Warranty Payments Case Study

MAS II Fall 2019

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B. Setup Study and Create Exploratory Data Output

In this case study, seven different models are displayed to model the costs for warranty payments for one product. All of the models displayed are Linear Mixed Effects models and use the nlme R package.

The product is sold in 100 different stores, but only three stores are included in the sample. Those three stores are under the variable Store and are denoted by: Store_1, Store_2 and Store_3.

The product was only sold for one year and the individual payments are recorded by month of sale,Sale_MOnth, and the month of payment after the sale. The warranty expires at the end of 12 months. The month of payment is denoted by number of months since the sale. For example, payments made on the warranty for products sold in January that were paid in January would show a payment lag, Pay_Lag, of 1. Payments for products sold in the month of February that were made in March would show a payment lag of 2.

The individual payments were transformed by using the natural logarithm transformation before the start of the analysis and are recorded in the column Log_Payment. The variable standard_log_payment is the result of normalizing the Log_Payment variable.

Set up Analysis by Loading Packages and Reading in Data Set

```
## -- Attaching packages -----------------------------------------------------
---------------------------------- tidyverse 1.2.1 --

## v ggplot2 3.2.0     v purrr   0.3.2
## v tibble  2.1.3     v dplyr   0.8.1
## v tidyr   0.8.3     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.4.0

## -- Conflicts --------------------------------------------------------------
---------------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

##
## Attaching package: 'nlme'

## The following object is masked from 'package:dplyr':
##
##     collapse

## New names:
## * Sigma_Store -> Sigma_Store...10
## * Sigma_Store -> Sigma_Store...12
```

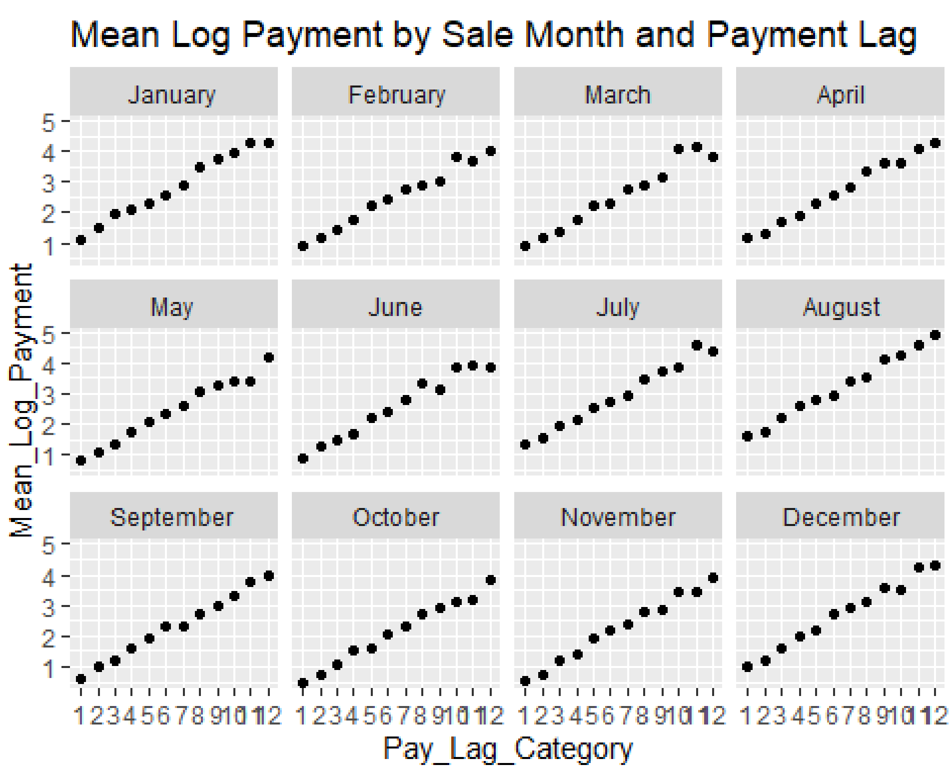
Exploratory Data Information

Summary information on the by claim data set as well as a series of graphs are displayed in this section to inform interpretation of the analysis that follows.

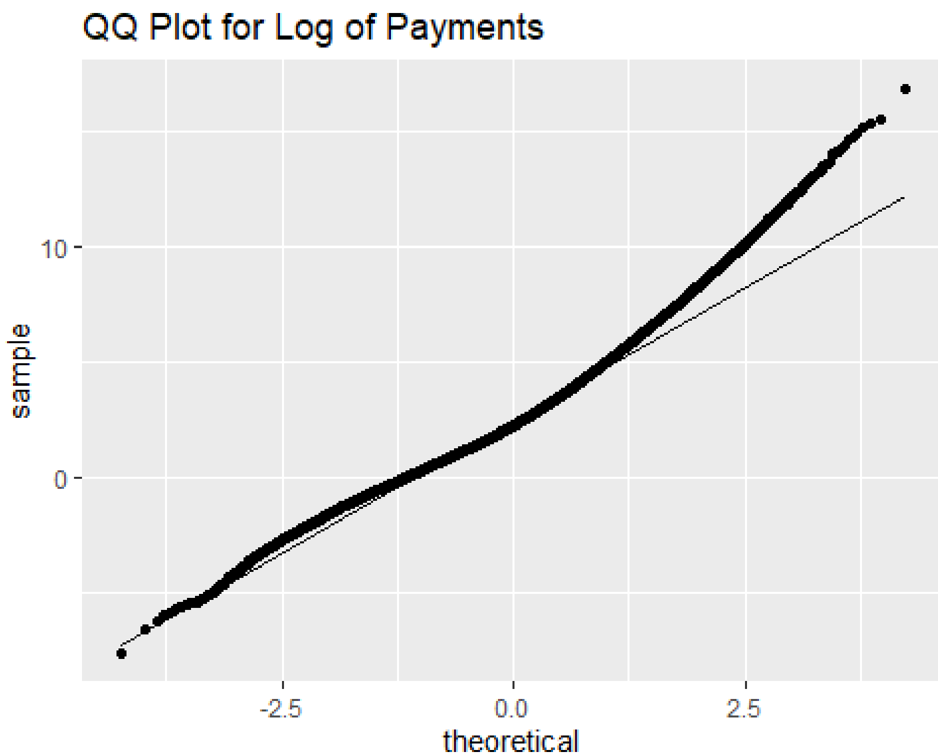
```
summary(Repair)

##  sale_month_lookup    Sale_Month       Store              Pay_Lag
##  Min.   : 1.0       December: 3601   Length:43200       Min.   : 1.0
##  1st Qu.: 4.0       February: 3600   Class :character   1st Qu.: 4.0
##  Median : 7.0       March   : 3600   Mode  :character   Median : 7.0
##  Mean   : 6.5       April   : 3600                      Mean   : 6.5
##  3rd Qu.:10.0       May     : 3600                      3rd Qu.:10.0
##  Max.   :12.0       June    : 3600                      Max.   :12.0
##                     (Other) :21599
##   Log_Payment          Payment          Pay_Lag_Category
##  Min.   :-7.6053   Min.   :       0    12      : 3601
##  1st Qu.: 0.9118   1st Qu.:       2    2       : 3600
##  Median : 2.2680   Median :      10    3       : 3600
##  Mean   : 2.5933   Mean   :    2199    4       : 3600
##  3rd Qu.: 4.0151   3rd Qu.:      55    5       : 3600
##  Max.   :16.8512   Max.   :20815028    6       : 3600
##                                        (Other):21599
```

```
##  standard_log_payment
##  Min.   :-4.1539
##  1st Qu.:-0.6849
##  Median :-0.1325
##  Mean   : 0.0000
##  3rd Qu.: 0.5791
##  Max.   : 5.8072
##
```

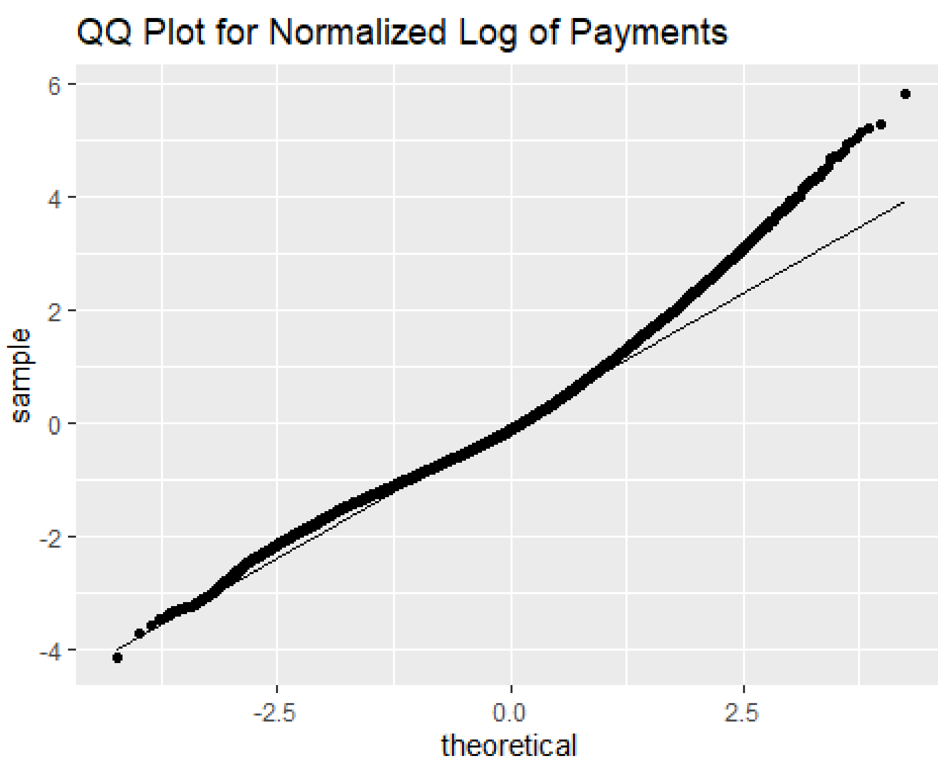
```
Repair_Group <- Repair %>% group_by(Sale_Month,Pay_Lag_Category) %>%
                mutate(Mean_Log_Payment =mean(Log_Payment),Std_Dev_Log_Paymen
t =sd(Log_Payment))

ggplot (data= Repair_Group, aes(x=Pay_Lag_Category, y= Mean_Log_Payment)) +ge
om_point() +
          facet_wrap(vars(Sale_Month)) +labs(title ="Mean Log Payment by Sale
Month and Payment Lag")
```



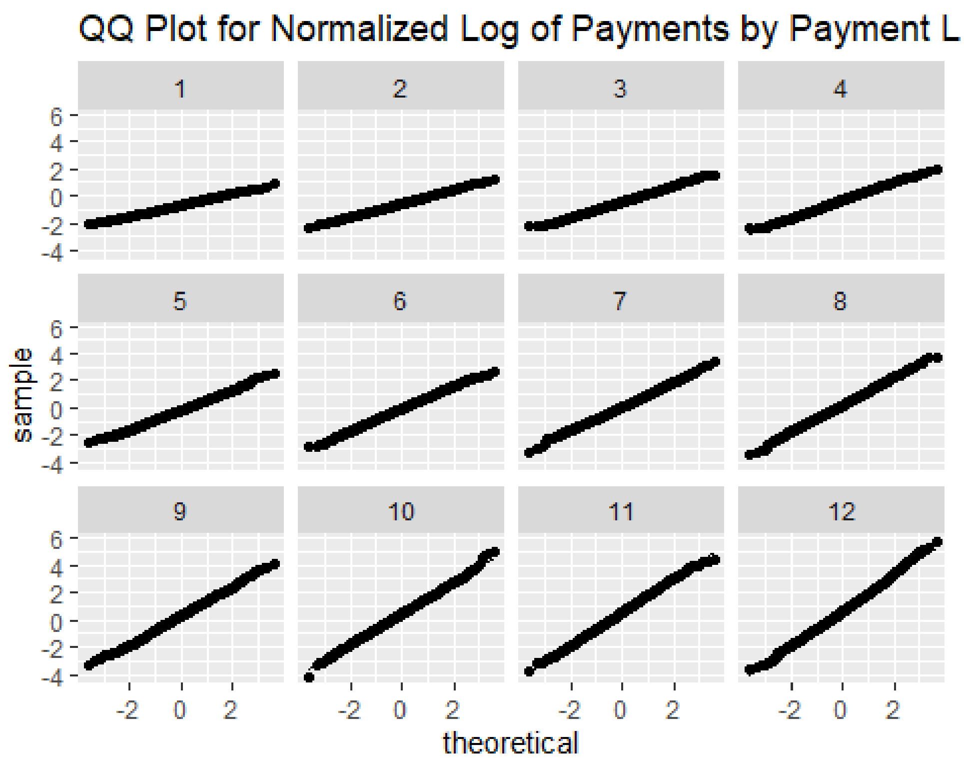

```
ggplot(data=Repair,aes(sample=Log_Payment)) + geom_qq() +geom_qq_line()+
  labs(title="QQ Plot for Log of Payments")
```




```
ggplot(data=Repair,aes(sample=standard_log_payment)) + geom_qq() +geom_qq_lin
e()+
  labs(title="QQ Plot for Normalized Log of Payments")
```

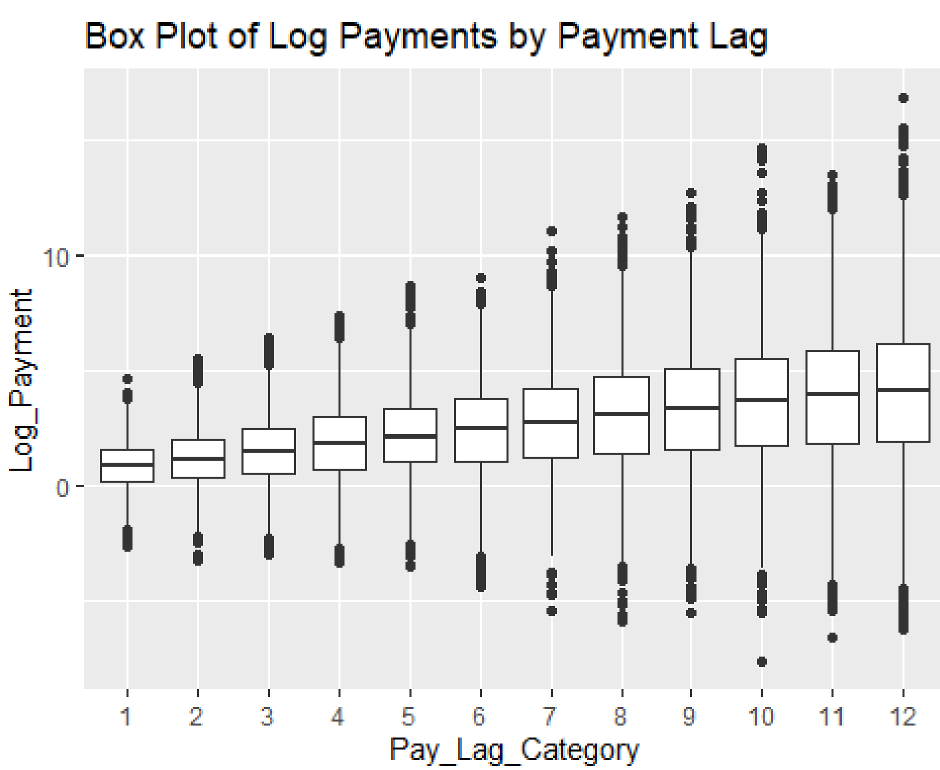


```
ggplot(data=Repair,aes(sample=standard_log_payment)) + geom_qq() +geom_qq_lin
e()+
  labs(title="QQ Plot for Normalized Log of Payments by Payment Lag") +
  facet_wrap(vars(Pay_Lag_Category))
```

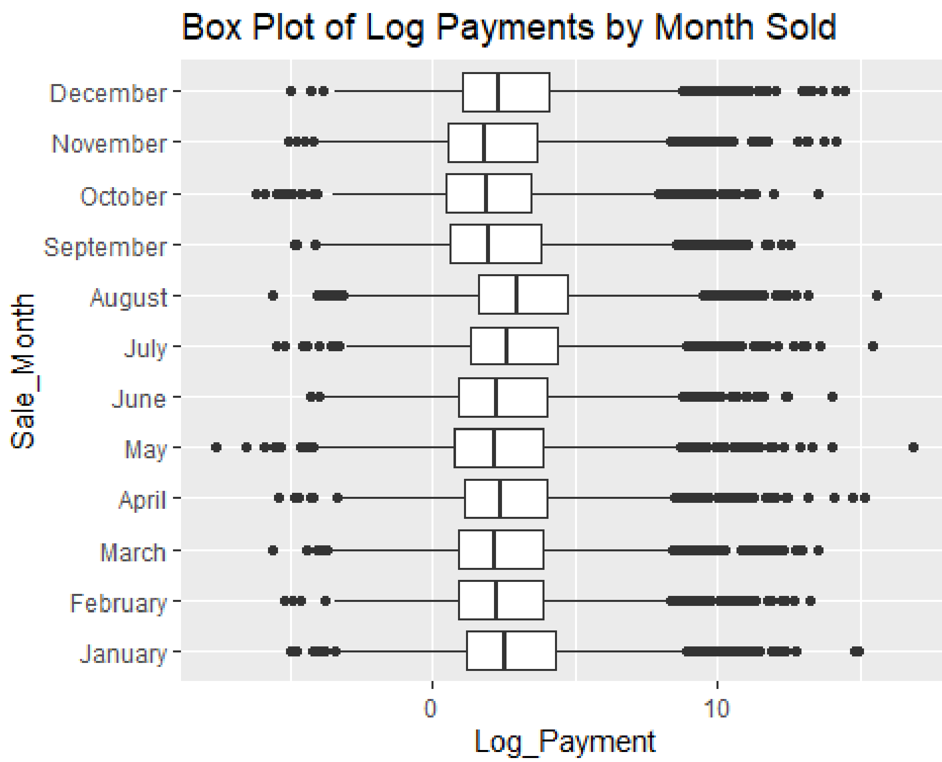



```
ggplot(data = Repair, aes(x=Store,y=Log_Payment))+ geom_boxplot()+
  labs(title="Box Plot of Log Payments by Store")
```


```r
ggplot(data = Repair, aes(x=Pay_Lag_Category,y=Log_Payment))+ geom_boxplot()+
  labs(title="Box Plot of Log Payments by Payment Lag")
```



```r
ggplot(data = Repair, aes(x=Sale_Month,y=Log_Payment))+ geom_boxplot()+
  labs(title="Box Plot of Log Payments by Month Sold")+ coord_flip()
```

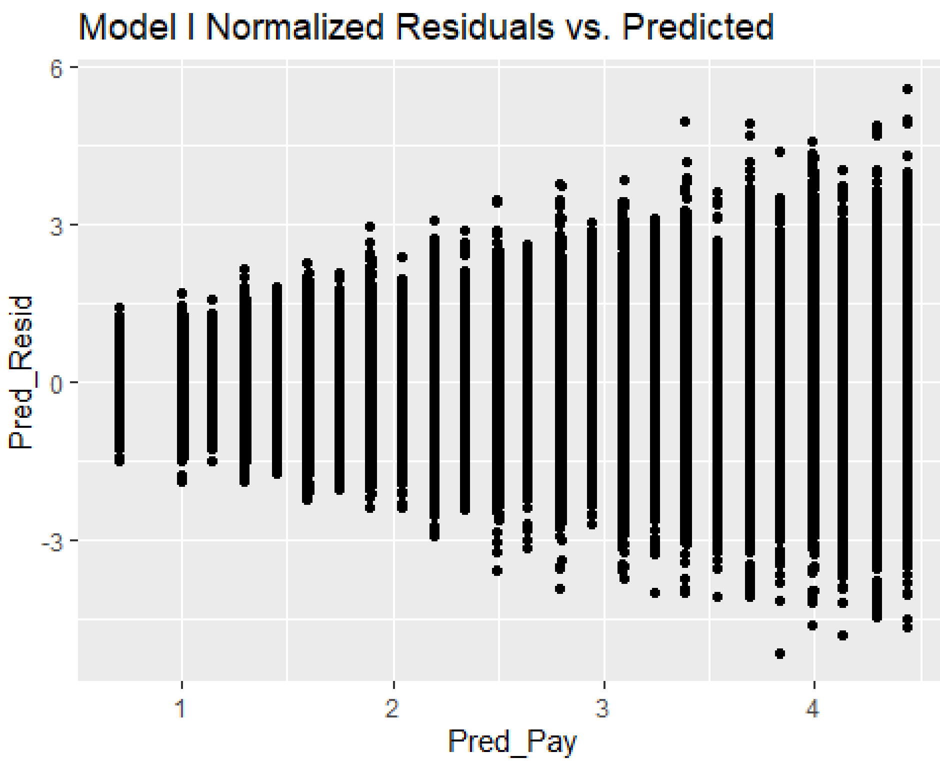


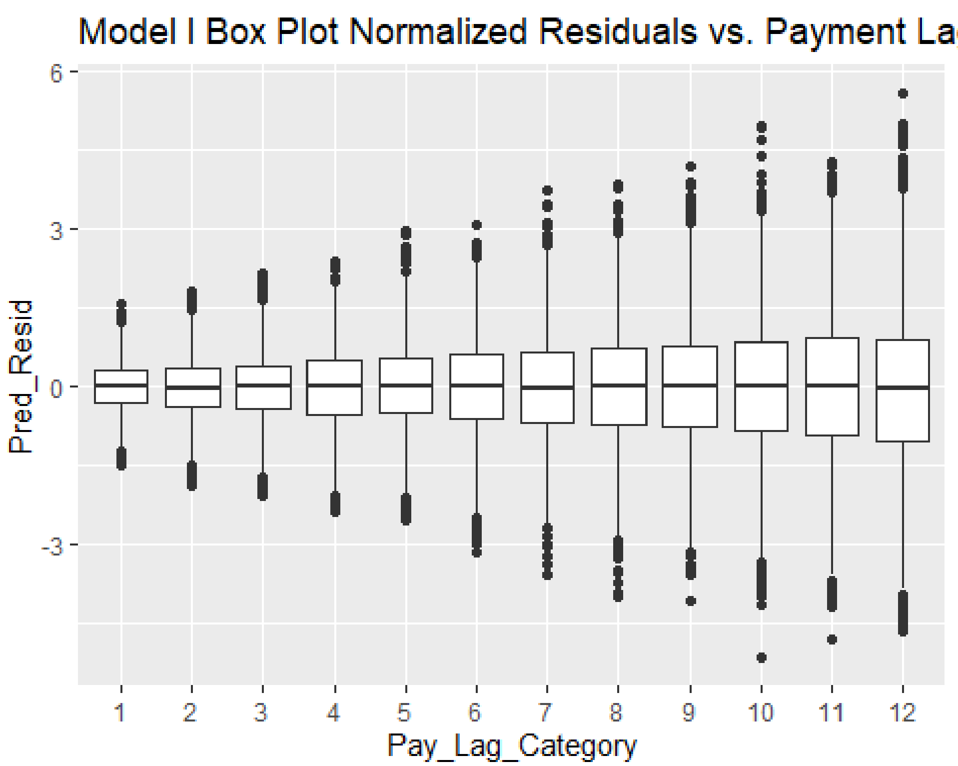
C. Results of Fitting Models

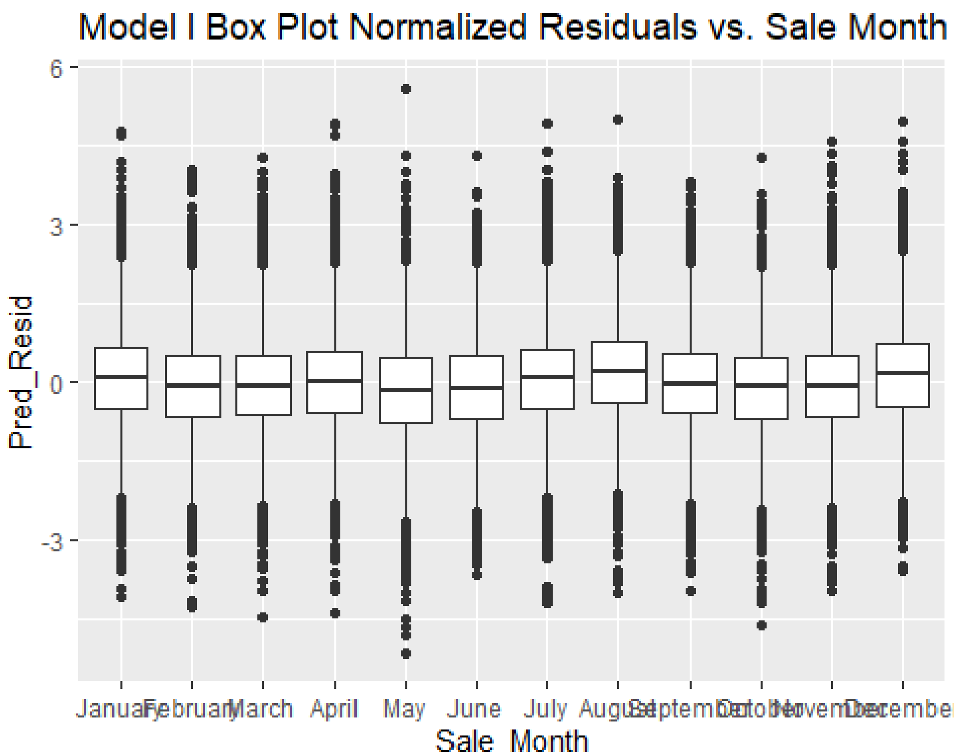
Create Model 1

Fixed effects include payment lag. Random effects include store. Variance is assumed to be constant.

```
## Linear mixed-effects model fit by REML
##  Data: Repair
##         AIC      BIC   logLik
##   191545.6 191580.3 -95768.8
##
## Random effects:
##  Formula: ~1 | Store
##         (Intercept) Residual
## StdDev:   0.2302979 2.220299
##
## Fixed effects: Log_Payment ~ Pay_Lag
##                 Value  Std.Error    DF  t-value p-value
## (Intercept) 0.6518274 0.13489914 43196  4.83196       0
## Pay_Lag     0.2986713 0.00309451 43196 96.51643       0
##  Correlation:
##         (Intr)
## Pay_Lag -0.149
##
## Standardized Within-Group Residuals:
##          Min           Q1          Med           Q3          Max
## -5.152857995 -0.587180201 -0.002330143  0.581518009  5.593045490
##
## Number of Observations: 43200
## Number of Groups: 3

##             numDF denDF  F-value p-value
## (Intercept)     1 43196  377.956  <.0001
## Pay_Lag         1 43196 9315.422  <.0001
```


Model I Normalized Residuals vs. Predicted
6
3
0
-3
Pred_Resid
1
2
3
4
Pred_Pay


Model I Box Plot Normalized Residuals vs. Payment La
6
3
0
-3
Pred_Resid
1
2
3
4
5
6
7
8
9
10
11
12
Pay_Lag_Category


Model I Box Plot Normalized Residuals vs. Sale Month
6
3
0
-3
Pred_Resid
January
February
March
April
May
June
July
August
September
October
November
December
Sale_Month

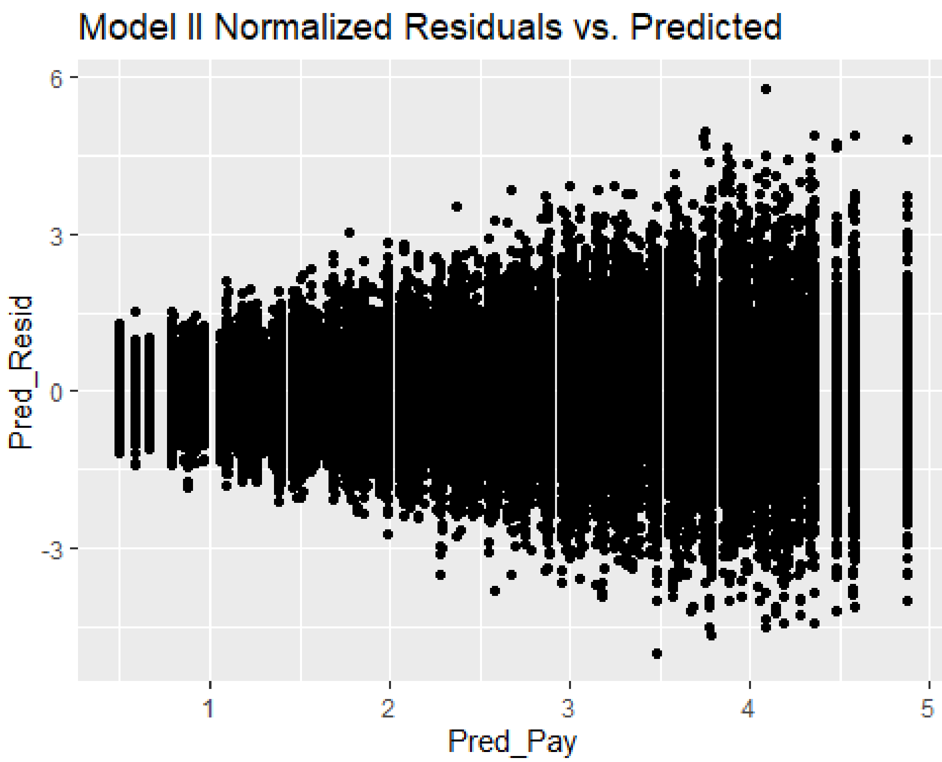
Create Model 2

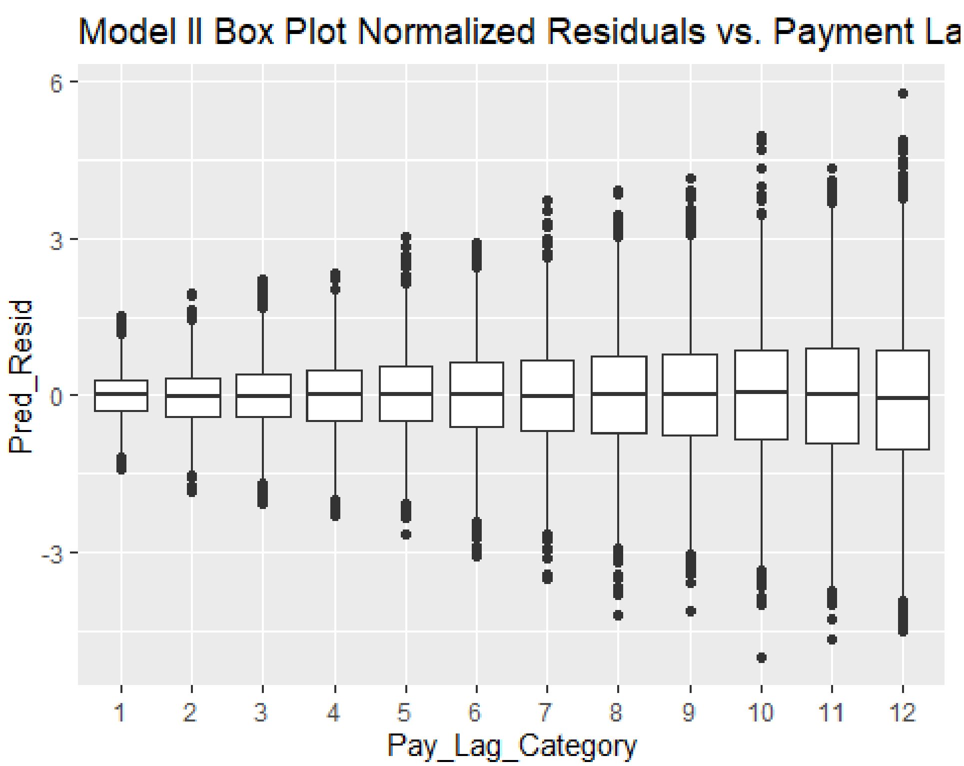
Fixed effects include payment lag and month sold. Random effects include store. Variance is assumed to be constant.

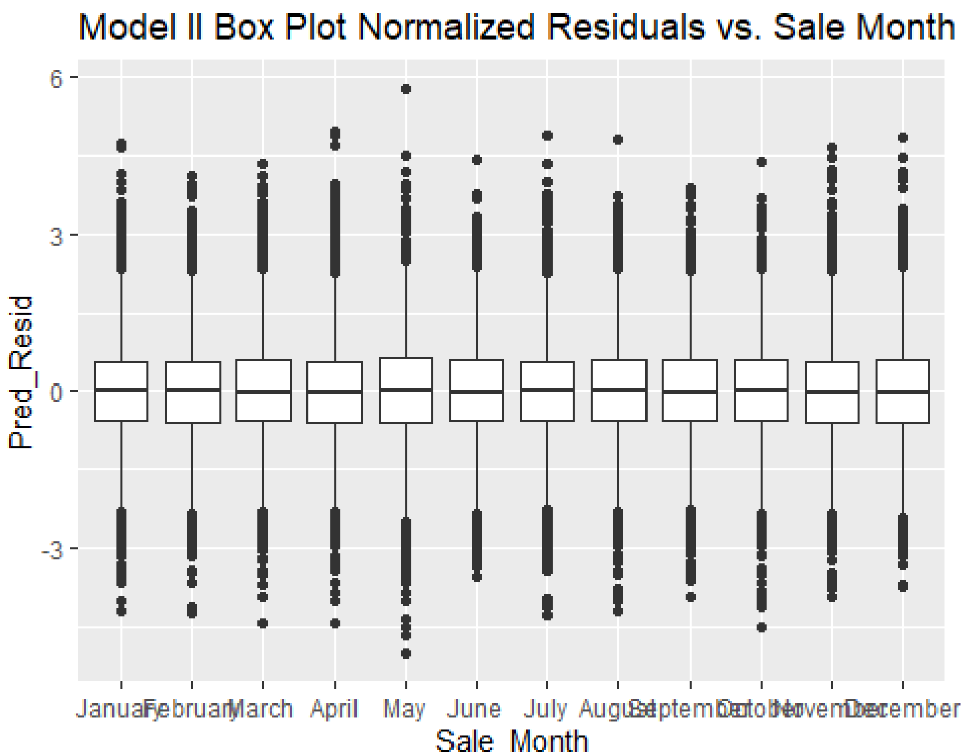
```
## Linear mixed-effects model fit by REML
##  Data: Repair
##        AIC      BIC    logLik
##   191136.2 191266.3 -95553.12
##
## Random effects:
##  Formula: ~1 | Store
##         (Intercept) Residual
## StdDev:  0.04907785 2.208503
##
## Fixed effects: Log_Payment ~ Pay_Lag + Sale_Month
##                         Value  Std.Error    DF  t-value p-value
## (Intercept)          0.8945594 0.06453183 43185 13.86230  0.0000
## Pay_Lag              0.2986655 0.00307807 43185 97.03005  0.0000
## Sale_MonthFebruary  -0.3342177 0.05205854 43185 -6.42004  0.0000
## Sale_MonthMarch     -0.2885262 0.05205854 43185 -5.54234  0.0000
## Sale_MonthApril     -0.1265996 0.05205854 43185 -2.43187  0.0150
## Sale_MonthMay       -0.3960827 0.08676038 43185 -4.56525  0.0000
## Sale_MonthJune      -0.2724004 0.08676038 43185 -3.13969  0.0017
## Sale_MonthJuly       0.1019019 0.08676038 43185  1.17452  0.2402
## Sale_MonthAugust     0.3891668 0.08676038 43185  4.48554  0.0000
## Sale_MonthSeptember -0.5303462 0.08676038 43185 -6.11277  0.0000
## Sale_MonthOctober   -0.7028963 0.08676038 43185 -8.10158  0.0000
## Sale_MonthNovember  -0.6114114 0.08676038 43185 -7.04713  0.0000
## Sale_MonthDecember  -0.1409673 0.08675821 43185 -1.62483  0.1042
##  Correlation:
##                     (Intr) Pay_Lg Sl_MnF Sl_MnthMr Sl_MnthAp Sl_MnthMy
## Pay_Lag             -0.310
## Sale_MonthFebruary  -0.403  0.000
## Sale_MonthMarch     -0.403  0.000  0.500
## Sale_MonthApril     -0.403  0.000  0.500  0.500
## Sale_MonthMay       -0.672  0.000  0.300  0.300     0.300
## Sale_MonthJune      -0.672  0.000  0.300  0.300     0.300     0.820
## Sale_MonthJuly      -0.672  0.000  0.300  0.300     0.300     0.820
## Sale_MonthAugust    -0.672  0.000  0.300  0.300     0.300     0.820
## Sale_MonthSeptember -0.672  0.000  0.300  0.300     0.300     0.500
## Sale_MonthOctober   -0.672  0.000  0.300  0.300     0.300     0.500
## Sale_MonthNovember  -0.672  0.000  0.300  0.300     0.300     0.500
## Sale_MonthDecember  -0.672  0.000  0.300  0.300     0.300     0.500
```

```
##                       Sl_MnthJn Sl_MnthJl Sl_MnthAg Sl_MnS Sl_MnO Sl_MnN
## Pay_Lag
## Sale_MonthFebruary
## Sale_MonthMarch
## Sale_MonthApril
## Sale_MonthMay
## Sale_MonthJune
## Sale_MonthJuly       0.820
## Sale_MonthAugust     0.820     0.820
## Sale_MonthSeptember  0.500     0.500     0.500
## Sale_MonthOctober    0.500     0.500     0.500     0.820
## Sale_MonthNovember   0.500     0.500     0.500     0.820  0.820
## Sale_MonthDecember   0.500     0.500     0.500     0.820  0.820  0.820
##
## Standardized Within-Group Residuals:
##          Min           Q1          Med           Q3          Max
## -5.021685091 -0.584161394 -0.002552077  0.579781948  5.781617861
##
## Number of Observations: 43200
## Number of Groups: 3
```

```
##             numDF denDF  F-value p-value
## (Intercept)     1 43185 7343.471  <.0001
## Pay_Lag         1 43185 9415.180  <.0001
## Sale_Month     11 43185   46.410  <.0001
```


Model II Normalized Residuals vs. Predicted
6
3
0
-3
Pred_Resid
1
2
3
4
5
Pred_Pay


Model II Box Plot Normalized Residuals vs. Payment La
6
3
0
-3
Pred_Resid
1
2
3
4
5
6
7
8
9
10
11
12
Pay_Lag_Category


Model II Box Plot Normalized Residuals vs. Sale Month
Pred_Resid
6
3
0
-3
January
February
March
April
May
June
July
August
September
October
November
December
Sale_Month

Create Model 3

Fixed effects include payment lag and month sold. Random effects include store. Variance is assumed to be a function of payment lag using the exponential variance function option.

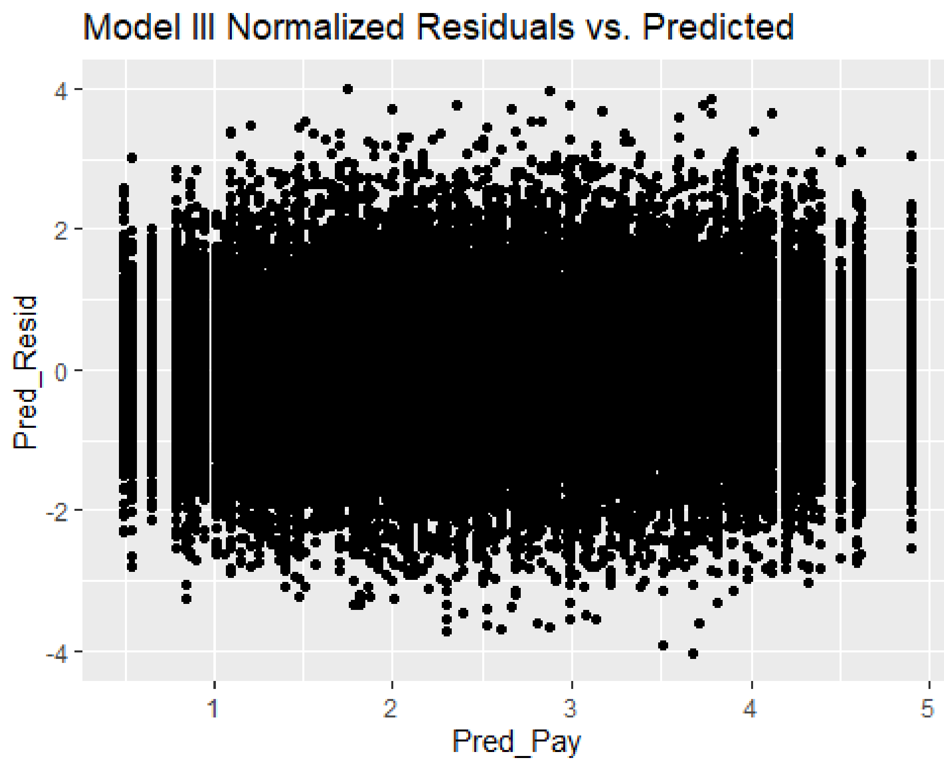
```
wgt_function_1 <- varExp(form = ~Pay_Lag/2)

Model_III <- lme (Log_Payment ~  Pay_Lag + Sale_Month, data =Repair,
                 random = ~1|Store,
                 weights=wgt_function_1)
summary(Model_III)

## Linear mixed-effects model fit by REML
##  Data: Repair
##        AIC      BIC    logLik
##   181627.8 181766.6 -90797.89
##
## Random effects:
##  Formula: ~1 | Store
##         (Intercept) Residual
## StdDev:  0.02249708 1.012199
##
## Variance function:
##  Structure: Exponential of variance covariate
##  Formula: ~Pay_Lag/2
##  Parameter estimates:
##     expon
## 0.2061607
## Fixed effects: Log_Payment ~ Pay_Lag + Sale_Month
##                          Value  Std.Error    DF   t-value p-value
## (Intercept)          0.8726579 0.03879148 43185  22.49612  0.0000
## Pay_Lag              0.3025872 0.00281533 43185 107.47826  0.0000
## Sale_MonthFebruary  -0.3083311 0.04133114 43185  -7.46002  0.0000
## Sale_MonthMarch     -0.3059846 0.04133114 43185  -7.40325  0.0000
## Sale_MonthApril     -0.1204501 0.04133114 43185  -2.91427  0.0036
## Sale_MonthMay       -0.3872547 0.05215842 43185  -7.42459  0.0000
## Sale_MonthJune      -0.2721021 0.05215842 43185  -5.21684  0.0000
## Sale_MonthJuly       0.1135696 0.05215842 43185   2.17740  0.0295
## Sale_MonthAugust     0.3874351 0.05215842 43185   7.42805  0.0000
## Sale_MonthSeptember -0.5276125 0.05215842 43185 -10.11558  0.0000
## Sale_MonthOctober   -0.6868433 0.05215842 43185 -13.16841  0.0000
## Sale_MonthNovember  -0.6349874 0.05215842 43185 -12.17421  0.0000
## Sale_MonthDecember  -0.1619186 0.05215784 43185  -3.10440  0.0019
```

```
##  Correlation:
##                      (Intr) Pay_Lg Sl_MnF Sl_MnthMr Sl_MnthAp Sl_MnthMy
## Pay_Lag              -0.310
## Sale_MonthFebruary   -0.533  0.000
## Sale_MonthMarch      -0.533  0.000  0.500
## Sale_MonthApril      -0.533  0.000  0.500  0.500
## Sale_MonthMay        -0.672  0.000  0.396  0.396     0.396
## Sale_MonthJune       -0.672  0.000  0.396  0.396     0.396     0.686
## Sale_MonthJuly       -0.672  0.000  0.396  0.396     0.396     0.686
## Sale_MonthAugust     -0.672  0.000  0.396  0.396     0.396     0.686
## Sale_MonthSeptember -0.672  0.000  0.396  0.396     0.396     0.500
## Sale_MonthOctober    -0.672  0.000  0.396  0.396     0.396     0.500
## Sale_MonthNovember   -0.672  0.000  0.396  0.396     0.396     0.500
## Sale_MonthDecember   -0.672  0.000  0.396  0.396     0.396     0.500
##                      Sl_MnthJn Sl_MnthJl Sl_MnthAg Sl_MnS Sl_MnO Sl_MnN
## Pay_Lag
## Sale_MonthFebruary
## Sale_MonthMarch
## Sale_MonthApril
## Sale_MonthMay
## Sale_MonthJune
## Sale_MonthJuly        0.686
## Sale_MonthAugust      0.686     0.686
## Sale_MonthSeptember   0.500     0.500     0.500
## Sale_MonthOctober     0.500     0.500     0.500     0.686
## Sale_MonthNovember    0.500     0.500     0.500     0.686  0.686
## Sale_MonthDecember    0.500     0.500     0.500     0.686  0.686  0.686
##
## Standardized Within-Group Residuals:
##          Min           Q1          Med           Q3          Max
## -4.029197902 -0.669171884 -0.002832774  0.662012553  3.994009865
##
## Number of Observations: 43200
## Number of Groups: 3
```

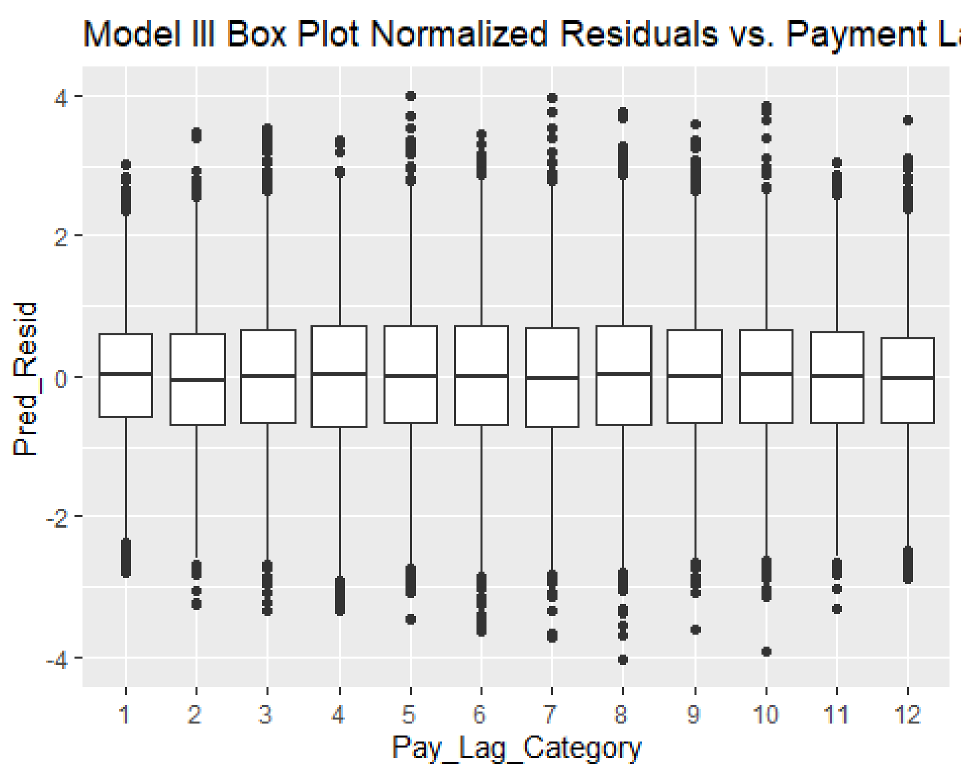
```
anova(Model_III)
```

```
##             numDF denDF   F-value p-value
## (Intercept)     1 43185 15381.830  <.0001
## Pay_Lag         1 43185 11551.994  <.0001
## Sale_Month     11 43185    79.906  <.0001
```

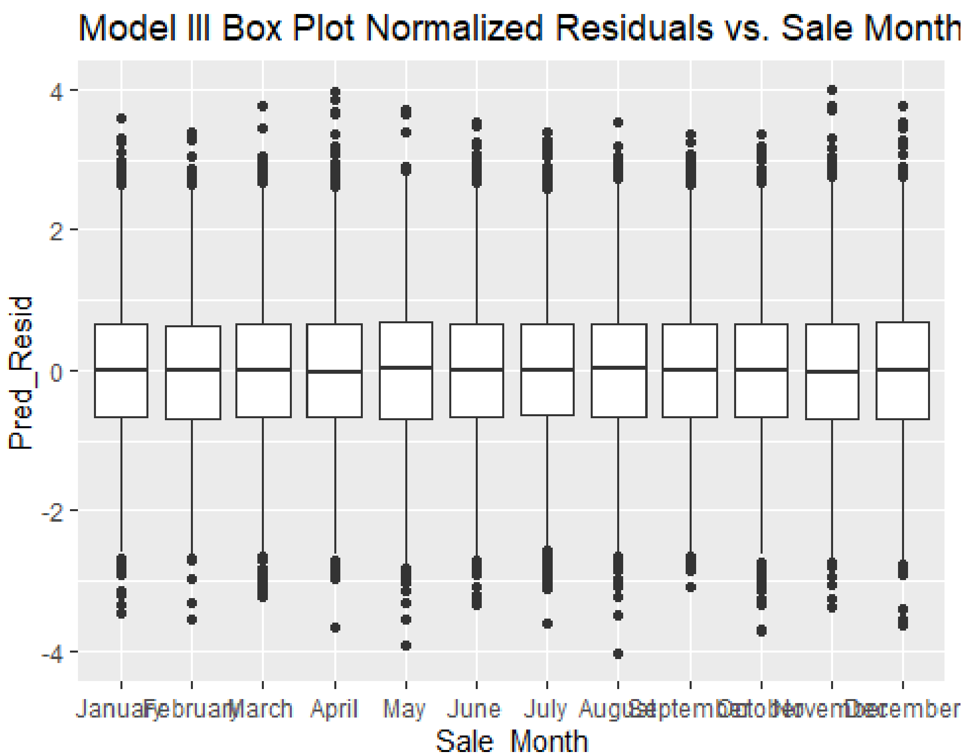
```
Model_III_Results <- Repair %>% mutate(Pred_Pay =fitted.values(Model_III), Pr
ed_Resid =residuals(Model_III,type= "normalized"))

ggplot( data=Model_III_Results, aes(x=Pred_Pay, y=Pred_Resid)) +geom_point()
+
  labs( title ="Model III Normalized Residuals vs. Predicted")
```




```
ggplot( data=Model_III_Results, aes(x=Pay_Lag_Category, y=Pred_Resid)) +geom_
boxplot() +
  labs( title ="Model III Box Plot Normalized Residuals vs. Payment Lag")
```



```
ggplot( data=Model_III_Results, aes(x=Sale_Month, y=Pred_Resid)) +geom_boxplo
t() +
  labs( title ="Model III Box Plot Normalized Residuals vs. Sale Month")
```



Create Model 4

Fixed effects include payment lag and month sold. Random effects include store. Variance is assumed to be a function of payment lag using the power transform of payment time option.

```
wgt_function_3 <- varPower(form =~Pay_Lag)

Model_IV <- lme (Log_Payment ~  Pay_Lag + Sale_Month, data =Repair,
                random = ~1|Store,
                weights=wgt_function_3)
summary(Model_IV)
```

```
## Linear mixed-effects model fit by REML
##  Data: Repair
##        AIC      BIC    logLik
##   181693.1 181831.8 -90830.53
##
## Random effects:
##  Formula: ~1 | Store
##         (Intercept)  Residual
## StdDev:  0.01945993 0.8774883
##
## Variance function:
##  Structure: Power of variance covariate
##  Formula: ~Pay_Lag
##  Parameter estimates:
##     power
## 0.4884623
## Fixed effects: Log_Payment ~ Pay_Lag + Sale_Month
##                          Value  Std.Error    DF   t-value p-value
## (Intercept)          0.8701807 0.03589243 43185  24.24413  0.0000
## Pay_Lag              0.3013128 0.00255453 43185 117.95252  0.0000
## Sale_MonthFebruary  -0.2967367 0.04020954 43185  -7.37976  0.0000
## Sale_MonthMarch     -0.2965901 0.04020954 43185  -7.37611  0.0000
## Sale_MonthApril     -0.1041056 0.04020954 43185  -2.58908  0.0096
## Sale_MonthMay       -0.3808330 0.04872560 43185  -7.81587  0.0000
## Sale_MonthJune      -0.2711228 0.04872560 43185  -5.56428  0.0000
## Sale_MonthJuly       0.1276205 0.04872560 43185   2.61917  0.0088
## Sale_MonthAugust     0.3969035 0.04872560 43185   8.14569  0.0000
## Sale_MonthSeptember -0.5260494 0.04872560 43185 -10.79616  0.0000
## Sale_MonthOctober   -0.6828486 0.04872560 43185 -14.01416  0.0000
## Sale_MonthNovember  -0.6354767 0.04872560 43185 -13.04195  0.0000
## Sale_MonthDecember  -0.1577172 0.04872484 43185  -3.23690  0.0012
```

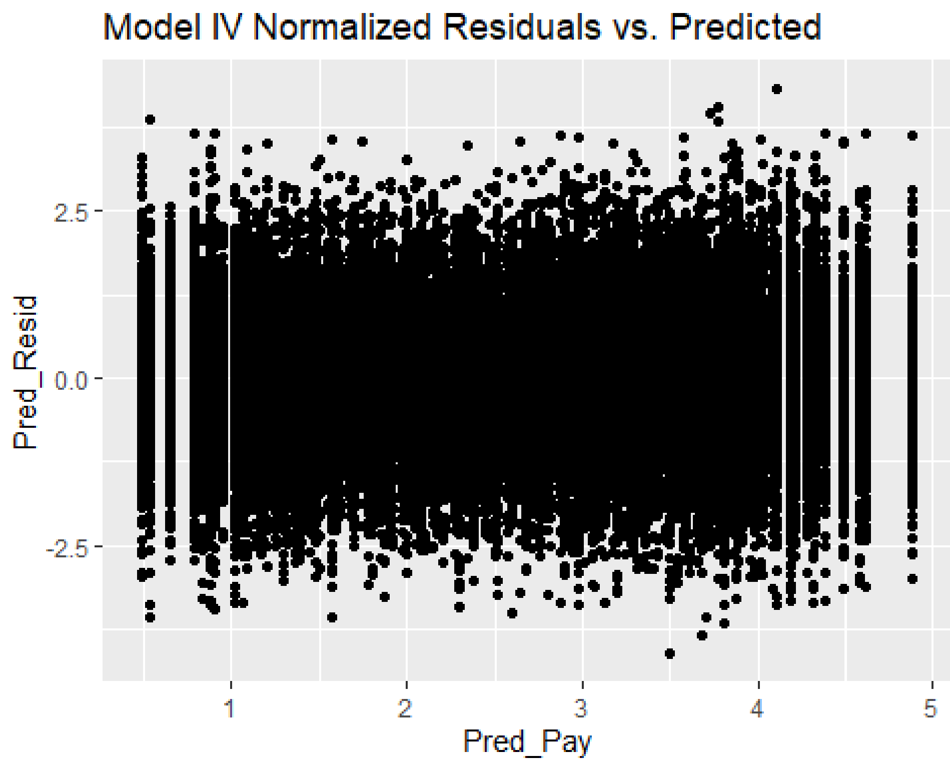
```
##  Correlation:
##                      (Intr) Pay_Lg Sl_MnF Sl_MnthMr Sl_MnthAp Sl_MnthMy
## Pay_Lag              -0.280
## Sale_MonthFebruary   -0.560  0.000
## Sale_MonthMarch      -0.560  0.000  0.500
## Sale_MonthApril      -0.560  0.000  0.500  0.500
## Sale_MonthMay        -0.679  0.000  0.413  0.413     0.413
## Sale_MonthJune       -0.679  0.000  0.413  0.413     0.413     0.660
## Sale_MonthJuly       -0.679  0.000  0.413  0.413     0.413     0.660
## Sale_MonthAugust     -0.679  0.000  0.413  0.413     0.413     0.660
## Sale_MonthSeptember -0.679  0.000  0.413  0.413     0.413     0.500
## Sale_MonthOctober    -0.679  0.000  0.413  0.413     0.413     0.500
## Sale_MonthNovember   -0.679  0.000  0.413  0.413     0.413     0.500
## Sale_MonthDecember   -0.679  0.000  0.413  0.413     0.413     0.500
##                      Sl_MnthJn Sl_MnthJl Sl_MnthAg Sl_MnS Sl_MnO Sl_MnN
## Pay_Lag
## Sale_MonthFebruary
## Sale_MonthMarch
## Sale_MonthApril
## Sale_MonthMay
## Sale_MonthJune
## Sale_MonthJuly       0.660
## Sale_MonthAugust     0.660     0.660
## Sale_MonthSeptember  0.500     0.500     0.500
## Sale_MonthOctober    0.500     0.500     0.500     0.660
## Sale_MonthNovember   0.500     0.500     0.500     0.660  0.660
## Sale_MonthDecember   0.500     0.500     0.500     0.660  0.660  0.660
##
## Standardized Within-Group Residuals:
##          Min           Q1          Med           Q3          Max
## -4.110764166 -0.668393442 -0.001169045  0.662217363  4.315151676
##
## Number of Observations: 43200
## Number of Groups: 3
```

```
anova(Model_IV)
```

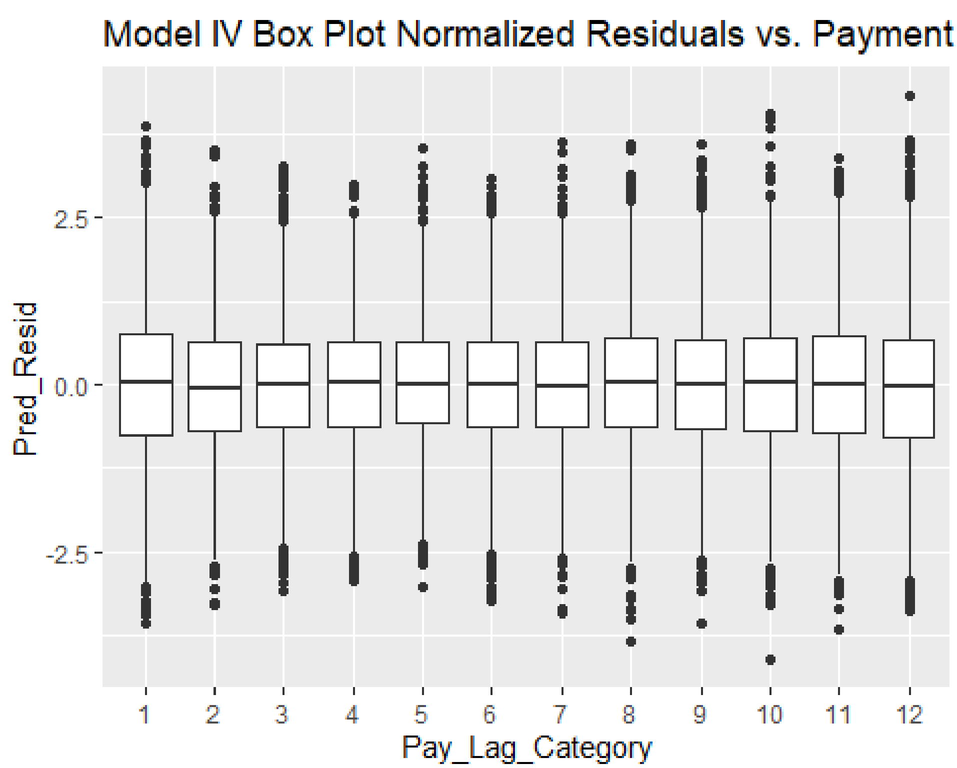
```
##             numDF denDF   F-value p-value
## (Intercept)     1 43185 17069.147  <.0001
## Pay_Lag         1 43185 13913.386  <.0001
## Sale_Month     11 43185    88.186  <.0001
```

```
Model_IV_Results <- Repair %>% mutate(Pred_Pay =fitted.values(Model_IV), Pred
_Resid =residuals(Model_IV,type= "normalized"))
```

```
ggplot( data=Model_IV_Results, aes(x=Pred_Pay, y=Pred_Resid)) +geom_point() +
  labs( title ="Model IV Normalized Residuals vs. Predicted")
```




```
ggplot( data=Model_IV_Results, aes(x=Pay_Lag_Category, y=Pred_Resid)) +geom_b
oxplot() +
  labs( title ="Model IV Box Plot Normalized Residuals vs. Payment Lag")
```



```
ggplot( data=Model_IV_Results, aes(x=Sale_Month, y=Pred_Resid)) +geom_boxplot
() +
  labs( title ="Model IV Box Plot Normalized Residuals vs. Sale Month")
```