CAS Exam 8

Seminar Style Slides
2026 Edition

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These are slides that would be presented
at a seminar.

While these presentations are self-contained,
the page numbers and question numbers refer to
my study guide, sold separately.

The presentations are in the same order as the
sections of my study guide.
Use the bookmarks in the Navigation Panel in
order to help you find what you want.

Going through them all,
pausing to do the problems,
I estimate would take about 60 hours.

My solutions. $\Leftrightarrow$ Model solutions.

See actual candidate responses in the solutions to past exam questions posted by the CAS.

See the examples of graded papers posted by the CAS.

I have included no questions from 2011 or later exams 8, so that you can use these as practice exams. In some cases, I have written similar questions and instead included those in the slides.

If this is you first exam with essay questions, be sure to spend extra time looking at the examples of CAS graded papers.

You can abbreviate, use lists, leave out words, show only one of a series of calculations, etc.

Write enough so the grader can easily tell that you know the answer.

Writing too much wastes valuable time.

Writing too little loses points.

Aim for somewhere in the middle.

Look at the points for a question.

The more points,
the more detailed explanation they expect.

Read the article on the CAS Webpage under Admissions:
"The Importance of Adverbs on Exams"

Briefly Define

Discuss

Fully Discuss

Do some past exam problems,
and have another student grade your paper.

At the beginning of my study guide is a grid of where the past exam questions have been.

This may help you to direct your study efforts.

More recent exams are more closely correlated with what will be on your exam.

You should concentrate a little more on what has been asked recently, but you still want to study the whole syllabus.
Just because something has not been asked for a few years does not mean it won't be asked on your exam.

The CAS will no longer be releasing past exams.

Make sure to study with the materials that will be attached to your exam, up to date version:

National Council on Compensation Insurance, **Experience Rating Plan** Manual for Workers Compensation and Employers Liability Insurance

Insurance Services Office, Inc., **Commercial General Liability Experience** and Schedule **Rating Plan**.

National Council on Compensation Insurance, **Retrospective Rating Plan** for Workers Compensation and Employers Liability Insurance.

Exam 8 will be given via
computer based testing.

Be sure to practice with the Excel-like spreadsheet you will be using.

Some overlap with the
CAS Basic Ratemaking Exam.

It may help to briefly review some of your notes on that exam about experience rating, retrospective rating, and large deductible policies.

Everything you need to know about these subjects for this exam should be in the relevant sections of my study guide.

Whatever study methods worked for you on earlier exams will probably work here.

Be flexible, you may have to tweak something here and there in studying for this exam.

Emphasize really understanding the material.

Do not emphasize shortcuts.

Know how to do calculations using important formulas.

Don't do all the problems from a given reading all at once.

Read the paper and the section in my study guide, and then do some problems.

Come back and do a few more problems in a few weeks.

Repeat.

Bloom's Taxonomy

There is no firm dividing line between levels.
The CAS, particularly on the Fellowship Exams, has been testing at the higher levels.

Integrative Questions (IQs) will differ from a typical exam question in three significant ways.

1. An IQ will be worth more points.
   One IQ could be worth 10-15% of the total exam.

2. Each IQ will require candidates to draw from multiple syllabus learning objectives
   in order to answer the question.

3. IQs will test at a higher average Bloom's Taxonomy level than a standard exam question.

The 2017 exam had one Integrative Question, while the 2018 and 2019 exams each had two Integrative Questions.

Section 1

An Example of Credibility and Shifting Risk Parameters
by Howard C. Mahler

Page 10.

Shifting risk parameters: The parameters defining the risk process for an individual insured are not constant over time. There are (a series of perhaps small) permanent changes to the insured's initial risk process as one looks over several years.

The private passenger automobile insurance experience of a town relative to the rest of the state, in other words the town's relativity, could shift as that town becomes more densely populated.

losing percentage of baseball team.
⇔ loss ratio of an insured (or class).

losing percentage of team compared to average.
⇔ loss ratio of an insured compared to average.
⇔ relativity of a class.

predicting future losing percentage of a team.
⇔ experience rating an insured.
⇔ determining new class relativity.

1. Insurance applications of credibility are complex, since different size risks have different degrees of partial credibility.
The baseball teams all play the same number of games; they are the same size, so there is no need for partial credibilities.

2. Insurance is complicated by loss development.
There is no loss development in baseball; when the season is over, we know the won-loss record.

3. An insurance portfolio changes over time, as new insureds are added and as old insureds leave. Mahler has the same baseball teams for 60 years.

The Meyers/Dorweiler criterion uses Kendall’s tau, a measure of correlation, which you are <u>not</u> required to know how to calculate for this exam.

The optimal credibility using the Meyers/Dorweiler criterion has a Kendall’s tau of 0.

We measure the correlation of:

$$\frac{\text{actual losing percentage}}{\text{predicted losing percentage}}, \text{ and}$$

$$\frac{\text{predicted losing percentage}}{\text{overall average losing percentage}}.$$

Item #1 is analogous to the modified loss ratio, the ratio of losses to modified premium.

Item #2 is analogous to the experience mod.

Thus **the Meyers/Dorweiler criterion desires that the correlation between the experience modification and the (subsequent) modified loss ratio be zero**.

After experience rating, all insureds should be equally desirable to underwriters.

1.31 (2 points)
Three experience rating plans are being compared.
You are trying to evaluate which is optimal.
Each rating plan has been tested on the same five different policies of similar size.
You compare the modification factor for each plan calculated before the policy period to the subsequent experience during the policy period.
The following tables summarize the indicated modifications and policy period experience.

Policy Number	Rating Plan 1 Modification Factor	Rating Plan 2 Modification Factor	Rating Plan 3 Modification Factor	Policy Period Experience
1	0.80	0.87	0.81	0.85
2	0.90	0.87	0.83	0.85
3	1.00	1.00	1.00	1.00
4	1.10	1.03	1.09	1.05
5	1.20	1.23	1.27	1.25

Which is the preferred plan based on the Meyers/Dorweiler criterion? Why?
Which is the preferred plan based on the least squared error criterion? Why?

For each set of predictions we calculate the errors: predicted - observed.

Policy Number	Rating Plan 1 Modification Factor	Error
1	0.80	-0.05
2	0.90	+0.05
3	1.00	0
4	1.10	+0.05
5	1.20	-0.05

Policy Number	Rating Plan 2 Modification Factor	Error
1	0.87	+0.02
2	0.87	+0.02
3	1.00	0
4	1.03	-0.02
5	1.23	-0.02

Plan 2 has positive errors for debit risks and negative errors for credit risks.
The errors are negatively correlated with the experience modifications.

Policy Number	Rating Plan 3 Modification Factor	Error
1	0.81	-0.04
2	0.83	-0.02
3	1.00	0
4	1.09	+0.04
5	1.27	+0.02

Plan 3 has negative errors for credit risks and positive errors for debit risks.
The errors are positively correlated with the experience modifications.
In the case of Plan 1, the errors have a correlation close to zero with the experience modifications.
Thus **by the Meyers/Dorweiler criterion, we prefer Plan 1**.

Plan 1 has a larger average squared error than plan 3, which has a larger average squared error than plan 2.
Thus **by the least squared error criterion we prefer plan 2**.

Page 46 **Conclusions**:

When shifting parameters over time is an important phenomenon, older years of data should be given substantially less credibility than more recent years of data.

The more significant this phenomenon, the more important it is to minimize the delay in receiving the data that is to be used to make the prediction.

Three different criteria were examined that can be used to select the optimal credibility: least squares, limited fluctuation, and Meyers/Dorweiler.

In applications, one or more of these three criteria should be useful.

While the first two criteria are closely related, the third criterion can give substantially different results than the others.

Generally the mean squared error can be written as a second order polynomial in the credibilities.

The coefficients of this polynomial can be written in terms of the covariance structure of the data.

This in turn **allows one to obtain linear equation(s) which can be solved for the least squares credibilities in terms of the covariance structure**.

1.40. (9, 11/95, Q.10) (1 point) Which of the following are conclusions of Mahler in "An Example of Credibility and Shifting Risk Parameters"?

1. When parameter shift is present, the optimal credibility (based on least squares criterion) for the most recent available year of data increases as the delay in receiving the data increases.
2. Older years of data receive greater credibility when parameter shift is present than when it is not.
3. When parameter shift is present, use as many years of data as possible to maximize the accuracy of the prediction.

9, 11/95, Q.10. Statement 1 is backwards. As the delay in receiving data increases, its predictive value decreases and the credibility decreases.
Statement 2 is backwards.
Statement 3 is backwards. If one gives each year equal weight, as the number of years increases, eventually the accuracy will decrease. (If one determines separate optimal credibilities by year, as the number of years increases, eventually the accuracy will no longer increase significantly.)
Comment: Conclusions are those of Bizarro-Mahler on a planet opposite of the real world.

1.9. (1 point) Mahler in "An Example of Credibility and Shifting Risk Parameters," concludes that to predict baseball losing percentages, a reasonable method is to use three years of data with
$Z_1 = 10\%$, $Z_2 = 10\%$, $Z_3 = 55\%$, and the remaining weight to the grand mean.
A baseball team had the following record:
2005: won 67 games and lost 95 games.
2006: won 61 games and lost 101 games.
2007: won 66 games and lost 96 games.
Using the above method, in 2008, what is the predicted record for this team for its first 88 games?

1.9. The teams predicted losing percentage is:
(10%) {95 / (67 + 95)} + (10%) {101 / (61 + 101)} + (55%) {96 / (66 + 96)} + (25%) (50%) = 0.572.

Out of 88 games, this team is expected to lose:
(0.572)(88) = 50.3 games.

Therefore, the predicted record is about:
38 wins and 50 losses.

Comment: This data is for the Tampa Bay Rays.

Page 13. A Chi-Square Test is used in the paper (pages 235-236) to test whether or not risk parameters shift over time.

I discuss this in detail in a subsection.

This is the same Chi-Square Goodness-of-Fit test you learned on an earlier exam.

The paper (pages 237-239) also uses the correlations between years of data in order to whether or not risk parameters shift over time.

This is discussed in detail in a subsection of my section.

The conclusion using both tests is that for this data set the risk parameters are shifting relatively quickly over time.

Thus this is a useful set of data to use to investigate the impact of this phenomenon.

Table 5 from the Paper

	Correlations	
Years Separating Data	NL	AL
1	0.651	0.633
2	0.498	0.513
3	0.448	0.438
4	0.386	0.360
5	0.312	0.265
6	0.269	0.228
7	0.221	0.157
8	0.190	0.124

The correlations decline as the separation increases.
Years further apart are less correlated than years closer together.

Data from last year is more valuable to predict the coming year, than data from 5 years ago.

Thus the NCCI Experience Rating Plan, which assuming equal volume gives equal weight to each year of data, is an approximation to the theoretically most accurate plan.

Section 4 of the Paper

"The first question to be answered is whether there is any real difference between the experience of the different teams, or is the apparent difference just due to random fluctuations.

This is the fundamental question when considering the application of experience rating."

"Binomial Test"

Table 3 in the Paper

Team	1	2	3	4	5	6	7	8
NL	53.4	49.9	47.3	51.8	44.7	56.5	47.8	48.8

If the experience for each team were drawn from the same probability distribution, the results for each team would be much more similar.

A Binomial distribution with a 50% chance of losing, for 9000 games, has a variance of:
9000 (1/2) (1 - 1/2) = 2250.
This is a standard deviation of 47 games lost, or 47 / 9000 = 0.5% in losing percentage.

Thus if all the teams' results were drawn from the same distribution, using the Normal Approximation, approximately 95% of the teams would have an average losing percentage between 49% and 51%.

Thus if all the teams' results were drawn from the same distribution, approximately 95% of the teams would have an average losing percentage between 49% and 51%.

Team	1	2	3	4	5	6	7	8
NL	53.4	49.9	47.3	51.8	44.7	56.5	47.8	48.8

Team	1	2	3	4	5	6	7	8
AL	49.5	49.4	47.0	48.5	42.6	52.9	56.4	53.5

Only 3 of 16 teams have losing percentages in that range.

The largest deviation from the grand mean is 15 times the expected standard deviation if the teams all had the same underlying probability distribution.

"There can be no doubt that the teams actually differ.
It is therefore a meaningful question to ask whether a given team is better or worse than average.
A team that has been worse than average over one period of time is more likely to be worse than average over another period of time."

Thus this is a useful data set to use to investigate experience rating.

1.48 (9, 11/98, Q.25) (4 points) For the past 25 years, the Bermuda Captives have battled in the highly competitive Island Sunshine League. Their losses in each individual 100 game season are shown below, in five year intervals. Also shown below are the 25 year average losing percentages for each team in the Island Sunshine League. Each team played 100 games in each of the 25 years.

Bermuda Captives Loss Record	5 Year Subtotal
Seasons 1 - 5	160
Seasons 6 -10	170
Seasons 11 - 15	294
Seasons 16 - 20	330
Seasons 21 - 25	296

Team	25 Year Average Loss %
Bermuda Captives	50.0%
Barbados Bombers	60.0%
Jamaica White Sox	55.0%
Trinidad Hurricanes	45.0%
Cayman Cubs	40.0%

Critical Chi-Square statistic at 95% confidence level: 9.488
In Mahler's paper "An Example of Credibility and Shifting Risk Parameters," the author discusses three tests to perform on the data sets being observed. Use Mahler and the data above to answer the following questions.

9, 11/98, Q.25

a. (0.5 point) Mahler performs a test using the binomial distribution on the data set. What is the purpose of this test?

b. (0.75 point) Perform the binomial test at the 95% confidence level using the standard normal approximation, and give your conclusion of that test with respect to the above data.

c. (0.5 point) Mahler performs a chi-square test on the data set. What is the purpose of this test?

d. (0.75 point) Perform the chi-square test described by Mahler at the 95% confidence level, and give your conclusion of that test with respect to the above data. Show all work.

e. (0.5 point) Mahler performs a correlation test on the data set. What is the purpose of this test?

f. (1 point) Describe how one would perform the correlation test on the above data set. What would the likely conclusion be on the above data set?

9, 11/98, Q.25

a. To determine whether the data for each team was drawn from the same probability distribution. In other words, to determine whether an "inherent difference" in loss % exists between teams.

b. The variance in losing percentage in 2500 games would be: (0.5)(0.5) / 2500 = 0.0001.
standard deviation is: 1%.
If the data for each team was drawn from the same probability distribution, we would expect to see about 95% of the teams results between:
50% ± (2)(1%) = 48% to 52%.
In this case only 1 out of 5 teams is in that range.
(Two of the teams have losing percentages 5 standard deviations from average, while two team have losing percentages 10 standard deviations from average!)
Thus we conclude that the teams differ.

9, 11/98, Q.25

c. The purpose is to test whether risk parameters shift over time. In other words, determine whether inherent loss potential (L%) is shifting over time for each team.

d. The Bermuda Captives have an overall losing percentage of 50%.
The observed number of losses per 5 years for this team is: (5) (100) (50%) = 250.
(For this team this happens to also be the a priori mean.)
Chi-Square statistic is: $(160 - 250)^2 / 250 + (170 - 250)^2 / 250 + (294 - 250)^2 / 250 + (330 - 250)^2 / 250 + (296 - 250)^2 / 250 = 99.808$.
(Number of groups - 1 = 5 - 1 = 4 degrees of freedom.)

Since 99.808 > 9.488, we reject the null hypothesis at the 95% confidence level (5% significance level). We conclude that the risk parameters shift over time, at least for the Bermuda Captives.

9, 11/98, Q.25 e. The purpose is to test whether risk parameters shift over time.

f. For each year we have a vector of length 5 of losing percentages by team.
For the one year differential, we examine the correlation of the 24 sets of pairs of data separated by one year:
year 1 versus year 2, year 2 versus year 3, etc.
Mahler uses Kendall's tau to measure the correlation.
We take the average of these 24 correlations for the one year differential. We do the same for the two year differential, using the correlation of the 23 sets of pairs of data by two years. We take the average correlation for the two year differential. We do the similar calculation for the other differentials in years.
If the risk parameters do not shift over time, the average correlation should not differ significantly between the one year differential, two year differential, and so forth. If the risk parameters shift over time, the average correlation should be highest for the one year differential, second highest for the two year differential, and so forth.
Given the results of the Chi-Square Test for the Bermuda Captives, the likely conclusion of this test is that the risk parameters shift over time.

Page 11. Section 3.1 of the paper
Advantages of the Baseball Data

1. Over a very extended period of time there is a constant set of risks (teams).
In insurance, there are generally insureds who leave the data base and new ones that enter.

2. The loss data over this extended period of time are readily available, accurate and final.
In insurance, the loss data are sometimes hard to compile or obtain and are subject to possible reporting errors and loss development.

3. Each of the teams in each year plays roughly the same number of games.
Thus the loss experience is generated by risks of roughly equal "size."
Thus, in this example, one need <u>not</u> consider the dependence of credibility on size of risk.

1.23. In "An Example of Credibility & Shifting Risk Parameters," Mahler uses the following notation:
τ^2 = between variance.
C(k) = covariance for data of the same risk, k years apart = "within covariance"
C(0) = "within variance".

For a data set, you are given τ^2 = 5, C(0) = 50, C(1) = 10, C(2) = 8, C(3) = 6, and C(4) = 4.
One will be using least squares credibility, with the complement of credibility given to the grand mean and varying weights to each year of data.
In each case, determine the optimal credibilities to be assigned to each year of data.
(a) (1 point) Use data for Year 1 to Predict Year 2.
(b) (1 point) Use data for Year 1 to Predict Year 3.
(d) (2 points) Use data for Years 1 and 2 to Predict Year 3.

1.23. For two different years:

$\text{Cov}[X_i , X_j] = \tau^2 + C(|i - j|)$.

For example,

$\text{Cov}[X_1, X_3] = \tau^2 + C(2) = 5 + 8 = 13$.

For a single year of data, $\text{Cov}[X_i, X_i] = \text{Var}[X_i] = \tau^2 + C(0) = 5 + 50 = 55$.

A covariance matrix is:

Year 1	55	15	13	11	9
Year 2	15	55	15	13	11
Year 3	13	15	55	15	13
Year 4	11	13	15	55	15
Year 5	9	11	13	15	55

Call the years 1951, 1952, etc.

Year 1	55	15	13	11	9
Year 2	15	55	15	13	11
Year 3	13	15	55	15	13
Year 4	11	13	15	55	15
Year 5	9	11	13	15	55

$\sum Z_j \text{Cov}[X_i , X_j] = \text{Cov}[X_i, X_{N+\Delta}]$, where we are predicting year N + Δ, using years 1 to N.

(a) Using data for Year 1951 to Predict Year 1952, the equation is: $Z_1 \text{Cov}[X_1 , X_1] = \text{Cov}[X_1, X_2]$.

55 Z = 15. ⇒ Z = 15 / 55 = **27.3%**.

(b) Using data for Year 1951 to Predict Year 1953, the equation is: $Z_1 \text{Cov}[X_1 , X_1] = \text{Cov}[X_1, X_3]$.

55 Z = 13. ⇒ Z = 13 / 55 = **23.6%**.

$$\begin{array}{l} \text{Year 1951} \\ \text{Year 1952} \\ \text{Year 1953} \\ \text{Year 1954} \\ \text{Year 1955} \end{array} \begin{pmatrix} 55 & 15 & 13 & 11 & 9 \\ 15 & 55 & 15 & 13 & 11 \\ 13 & 15 & 55 & 15 & 13 \\ 11 & 13 & 15 & 55 & 15 \\ 9 & 11 & 13 & 15 & 55 \end{pmatrix}$$

$\sum Z_j \text{Cov}[X_i , X_j] = \text{Cov}[X_i, X_{N+D}]$, where we are predicting year N + D, using years 1 to N.

(d) Using data for Years 1951 and 1952 to Predict Year 1953, the equations are:

$$Z_1 \text{Cov}[X_1 , X_1] + Z_2 \text{Cov}[X_1 , X_2] = \text{Cov}[X_1, X_3].$$
$$Z_1 \text{Cov}[X_2 , X_1] + Z_2 \text{Cov}[X_2 , X_2] = \text{Cov}[X_2, X_3].$$

$$55 Z_1 + 15 Z_2 = 13.$$
$$15 Z_1 + 55 Z_2 = 15.$$

Year 1951	55	15	13	11	9
Year 1952	15	55	15	13	11
Year 1953	13	15	55	15	13
Year 1954	11	13	15	55	15
Year 1955	9	11	13	15	55

$55 Z_{1951} + 15 Z_{1952} = 13.$

$15 Z_{1951} + 55 Z_{1952} = 15.$

The coefficients on the lefthand side are the first two rows and the first two columns of the covariance matrix, since we are using data from Years 1951 and 1952.

The values on the righthand side are the first two rows of column three, since we are predicting year 1953.

Solving, $\mathbf{Z_{1951} = 17.5\%}$ and $\mathbf{Z_{1952} = 22.5\%}$.

Comment: See Equation 11.3 in Mahler.
The more recent, Year 1952, is given more weight than Year 1951, for predicting Year 1953.

With no delay in getting data, $\Delta = 1$,
similar to Mahler's Table 16:

	Years Between Data and Estimate		
Number of Years of Data Used (N)	1	2	3
1	27.3%		
2	22.5%	17.5%	
3	20.6%	15.0%	11.0%

With a delay in getting data, $\Delta = 2$:

	Years Between Data and Estimate		
Number of Years of Data Used (N)	2	3	4
1	23.6%		
2	19.6%	14.6%	
3	18.1%	12.7%	8.6%

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Not on the syllabus, the following is taken from "A Markov Chain Model of Shifting Risk Parameters", by Howard C. Mahler, PCAS 1997.

Define the "half-life" as the length of time it takes for the correlation between years of data to decline by a factor of 1/2.

The longer the half-life, the slower the rate of shifting risk parameters over time.

	Half-Life
Baseball Data	3.4 years
Female California Drivers	17.3 years

1.47. (9, 11/98, Q.14) (1 point) In "An Example of Credibility and Shifting Risk Parameters," Mahler discusses the maximum reduction in the mean squared error of an estimate that can be accomplished by using credibility.

You are given the following estimates based upon one year of data:

Mean squared error relying solely on the data = 80.

Mean squared error ignoring the data = 100.

What is the best mean squared error that can be achieved using a linear weighted average of the two estimates?

9, 11/98, Q.14.

The best that can be done using credibility to combine two estimates is to reduce the mean squared error between the estimated and observed values to 75% of the minimum of the squared errors from either relying solely on the data or ignoring the data.

(75%) (80) = **60**.

Comment: See Section 8.5 in the paper.

One can think of half of the squared error as being due to two sources: the inherent process variance associated with comparing to observed results, and the presence of shifting parameters over time.

This portion of the squared error is independent of the value chosen for the credibility.

The remainder of the squared error can be thought of as that which is affected by the choice of the value of credibility; this can be at most cut in half by the use of credibility methods.

If half of the squared error is cut in half, this reduces the total squared error to 75% of what it was.

1.22. (2 points) (For baseball fans) You are updating the study in Mahler's paper using similar baseball data from 1961 to the present.
(a) Mention two complications that would occur that Mahler did not have to deal with.
(b) Would you expect shifting risk parameters to have a bigger effect or smaller effect than in Mahler's study? Why?

A study question to help you think through the ideas.
<u>Not</u> an exam question, since you do not need to know the details of any particular sport or sports league.

1.22. (a) (1) Some new teams entered the leagues due to expansion. Mahler had the same 8 teams in each league throughout. We would have varying numbers of teams. For example, in 1969 the Kansas City Royals and Seattle Pilots (now the Milwaukee Brewers) joined the American League. These new teams were worse than average. Thus the existing teams seemed to improve on average between 1968 and 1969.

(2) Some seasons were shortened by strikes. Thus there are some years where a significantly smaller number of games were played.

(3) Leagues were split into divisions, and in recent seasons, teams play teams within their division more frequently. Thus unlike in Mahler's study, teams do not play approximately the same number of games against each other team in their league. If in a given season a certain division is significantly stronger than average, then the teams in that division play opponents who are stronger than average. Therefore, the expected winning percentages for teams in that division would be lower than it would otherwise be if there was a balanced schedule.

(4) Interleague play was introduced recently. While only about 10% of games involve play between the two leagues, this complication was not present in Mahler's Study.
The average winning percentage for a league is no longer 50% each year.
(For example, in 2006 the American League won 154 out of 252 interleague games; 154 / 252 = 61%. Thus that year, the average winning percentage for the American League was greater than 50%.)

Also the expected winning percentage of a team is effected by which teams it is scheduled to play that season.
Each season, a team only plays some of the teams in the other league and that varies from year to year.

(b) Since Mahler's study, free agency was introduced. Thus players switch teams more frequently now. Thus I would expect the effect of shifting risk parameters to be greater than in Mahler's study.

Alternately, the difference between the best and the worst teams is usually less than in Mahler's study; there is more parity among the teams. Therefore, there is a smaller region in which the winning percentages can vary from year to year. Thus I would expect the effect of shifting risk parameters to be less than in Mahler's study.

Alternately, since Mahler's study, baseball has instituted a draft. Teams with the worst record get to draft earlier. This will tend to allow bad teams to get better more quickly. Conversely good teams will have a harder time staying good for a long time. Therefore, parameters may shift more quickly than in the era in Mahler's study.

Comment: There are many possible reasonable answers.
In part (a) only give two reasons.
I have a similar question on the NFL for fans of American football.

1.50. (9, 11/00, Q.34) (2 points) Answer the following based on Mahler's "An Example of Credibility and Shifting Risk Parameters."

a. (1.5 points) Briefly describe three criteria used to compare the performance of credibility methods.

b. (0.5 points) Mahler states that one criterion differs from the other two criteria on a conceptual level. Which criterion is that? Briefly state in what way it differs from the others.

9, 11/00, Q.34

a. 1. Least squares - minimize the total squared error between actual and predicted result.

2. Small chance of large error - minimize the likelihood that any one actual observation will be a certain % different from the predicted result.

3. Meyers/Dorweiler - minimize the correlation between the ratio of actual/predicted and the predicted/average actual.

b. Meyers/Dorweiler is different from the first two which focus on minimizing prediction error.
In contrast, Meyers/Dorweiler focuses on the pattern of the errors.

Section 2

"An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car,"

by Robert A. Bailey and Leroy J. Simon

Including the Discussion by William J. Hazam

Bailey and Simon use Merit Rating data to determine the credibility to assign to the experience of a single private passenger car.

The most important parts of this concise paper are Tables 2 and 3, and their conclusions.

A key concept is that when using credibility,
Z is the discount compared to <u>average</u> given to an insured who is claims-free.

This credibility varies by class and the number of years claims-free.

Bailey-Simon compare a prior three year period to a subsequent one year period for Private Passenger Automobile Insurance in Canada.
The data is for PY1958 and PY1959.

They compare the subsequent frequency for groups with different numbers of years claims-free.

They found that Merit Rating has useful predictive ability beyond that of class and territory.

The then current Canadian Merit Rating Plan:

Those who are claim-free for only one year
get a discount of 10%, Group Y.

For example, Merit Rating a 1958 policy:
1957 claim free, but 1956 has a claim.

Those who are claim-free for only 2 years
get a discount of 20%, Group X.

For example, Merit Rating a 1958 policy:
1956 and 1957 claim free, but 1955 has a claim.

Those who are claim-free for 3 or more years
get a discount of 35%, Group A.

These discounts are off the base rate
for those who are not claims-free, Group B.

As stated at the first page of Bailey-Simon:

Earned premiums are converted to a common rate basis by use of the relationship in the rate structure that A: X: Y: B = 65: 80: 90: 100.

Bailey-Simon put premiums on the level
that would have been charged
for Merit Rating Class B,
those who are not claims free.

For example, if the actual premiums for
Merit Rating Group A were 6.5 million,
then on a Group B basis they would be:
6.5 / (1 - 35%) = 10 million.

In the context of credibility theory, actuaries are interested in the experience and discounts with respect to <u>average</u>.

<u>Table 1</u>:

We need to combine Groups A and X in order to get those who are claims free for 2 years or more. A + X + Y is those who are claims free for 1 year or more.

Class 2 - Pleasure - Non-principal male operator under 25				
Group	Years Claims-Free	Group B Premium	Number of Claims	Freq.
A	3 or more	11,840,000	14,506	1.225
A+X	2 or more	12,552,000	15,507	1.235
A+X+Y	1 or more	13,496,000	16,937	1.255
Total		15,488,000	20,358	1.314

For Class 2, the overall frequency on a <u>premium</u> basis is: 20,358 / 15,488 = 1.314.

The frequency on a <u>premium</u> basis for Group A (3 years claims-free) is: 14,506 / 11,840 = 1.225.

Bailey and Simon “have chosen to calculate Relative Claim Frequency on the basis of premium rather than car years.

This avoids the maldistribution created by having higher claim frequency territories produce more X, Y, and B risks and also produce higher territorial premiums.”

For Class 2, the overall frequency on a <u>premium</u> basis is: 20,358 / 15,488 = 1.314.

The frequency on a premium basis for Group A (3 years claims-free) is: 14,506 / 11,840 = 1.225.

Thus the relative claim frequency for Group A is: 1.225 / 1.314 = 0.932.

0.932 is the indicated experience modification for Group A.

0.932 is the indicated experience modification for Group A (three years claims free).

⇒ The claims free discount is: 1 - 0.932 = 6.8%.

This is the estimated credibility for three years of data shown in Table 2 for Class 2.

1 - Z = M =

$$\frac{\text{Prem. Based Claim Freq. Claims - Free N or More Years}}{\text{Overall Premium Based Claim Frequency for the Class}}.$$

Calculating in this manner the credibilities for one, two or three years is the most commonly asked exam question on this paper.

2.2ab You are given the following data on the Adult Drivers Class for P.P. Auto Liability.
Shown is the number of years they were without accident prior to 2010, the number of claims they had during 2010, and their loss cost premium during 2010 prior to the effects of Merit Rating:

Years since last accident	Premium ($ million)	Claims
5+	1520	134,200
4	70	8,900
3	80	10,400
2	90	12,500
1	100	14,400
0	140	19,600
Total	2000	200,000

a. (1 point) What is the credibility of 5 or more accident-free years of experience?
b. (1 point) What is the credibility of 4 or more accident-free years of experience?

2.2. The overall claim frequency on a premium basis is: 200,000 / 2000 = 100.

(a) Claim frequency on a premium basis for 5 or more years claim free: 134,200 / 1520 = 88.289.
1 - Z = 88.289 / 100. ⇒ Z = **11.7%**.

(b) Claim frequency on a premium basis for 4 or more years claim free:
(134,200 + 8900) / (1520 + 70) = 90.
1 - Z = 90 / 100. ⇒ Z = **10.0%**.

Comment: In part (b) those who have no claims in a 4 year window are:
those 4 years claims free
plus those claims free for 5 or more years.

P. 118 Ratio of Credibility to Frequency:

In addition, in Table 2,
for each class Bailey-Simon takes the ratio of the three-year credibility to the frequency.

For Class 2, the overall exposure based frequency is: 20.358 / 168,998 = 0.120.

The ratio of the 3-year credibility to frequency is: 0.068 / 0.120 = 0.567.

The credibilities depend on the Expected Value of the Process Variance (EPV) and
the Variance of the Hypothetical Means (VHM).

If each insured is Poisson, then the EPV is equal to the average frequency for the class.
In any case, the EPV should be roughly proportional to the mean frequency.

If the Buhlmann Credibility formula holds,
then the three-year credibility is:
$Z = 3 / (3 + K)$, with $K = EPV / VHM$.

For K big compared to 3,
$Z \cong 3 / K = (3)(VHM / EPV)$.

Let μ be the overall mean frequency, which is also the mean of the hypothetical mean frequencies. Assume the EPV is (approximately) proportional to the overall mean frequency: $EPV = c\,\mu$.

Then the ratio of the credibility to the mean frequency is approximately:
$(3)(VHM / EPV) / \mu = (3/c)\, VHM / \mu^2$.

Thus the ratio of the credibility to the mean frequency is proportional to the square of the coefficient of variation of the hypothetical means: VHM / μ^2.
Thus the smaller this ratio, the smaller the CV of the hypothetical means, and the less variation between the insureds within a class.

Thus the smaller this ratio of credibility to frequency, the more homogeneous the class.

As shown in Table 2 of Bailey-Simon:

Class	Three-Year Credibility	Claim frequency per car-year	Ratio
1	8.0%	8.7%	0.920
2	6.8%	12.0%	0.567
3	8.0%	14.2%	0.563
4	9.9%	16.2%	0.611
5	5.9%	11.0%	0.536

With the highest ratio,
Class 1 is the least homogeneous,
in other words the most heterogeneous.

I would <u>not</u> memorize the definitions of the classes in Bailey-Simon:

Class 1: Pleasure - no male operator under 25.

Class 2:
Pleasure - Non-principal male operator under 25.

Class 3 is Business use.

Class 4:
Unmarried owner or principal operator under 25.

Class 5:
Married owner or principal operator under 25.

Class	Three-Year Credibility	Claim frequency per car-year	Ratio
1	8.0%	8.7%	0.920
2	6.8%	12.0%	0.567
3	8.0%	14.2%	0.563
4	9.9%	16.2%	0.611
5	5.9%	11.0%	0.536

“Classes 2, 3, 4 and 5 are more narrowly defined than Class 1, and the fact that the ratios in the last column of Table 2 for these classes are less than the ratio for Class 1 confirms the expectation that there is less variation of individual hazards in those classes.”

Class 1: Pleasure - no male operator under 25.

This also illustrates that
credibility for experience rating depends not only on the volume of data in the experience period but also on the amount of variation of individual hazards within the class."

2.15. (2 points) Determine which of the current classes exhibits less variation of individual hazards than the others.
Use the data shown below:

	Claim Frequency per $1,000 Earned Premium	Earned Premium per Earned Car Year	Credibility of 3 years of Data from a Single Car
Class 1	0.263	$300	5.8%
Class 2	0.369	$400	9.3%
Class 3	0.311	$350	8.1%

Assume that the earned premiums are adjusted to a common current rate level.
Show all work.

2.15. For each class, we get the frequency per exposure by multiplying the frequency per $ premium times the premium per exposure.
For example, for Class 1:

(0.000263)(300) = 7.89%.

Then take the ratio of the 3-year credibility to this frequency, as per Table 2 in Bailey-Simon.
For example, for Class 1: 5.8% / 7.890% = 0.7351.

Class	Cred.	Class Freq. per Prem.	Prem. per Expos.	Freq. per Expos.	Cred. / Freq.
1	5.8%	0.000263	300	7.890%	**0.7351**
2	9.3%	0.000369	400	14.760%	**0.6301**
3	8.1%	0.000311	350	10.885%	**0.7441**

A more homogeneous class will have a ratio of credibility for experience rating to frequency that is lower. Thus Class 2 is more homogeneous than Classes 1 and 3;
Class 2 exhibits less variation of individual hazards than do the others.

<u>Comment</u>: Similar to 9, 11/95, Q.32.

P. 120 **Table 3**:

As shown in Table 2 of Bailey-Simon:

Class	One-Year Credibility	Two-Year Credibility	Three-Year Credibility
1	4.6%	6.8%	8.0%
2	4.5%	6.0%	6.8%
3	5.1%	6.8%	8.0%
4	7.1%	8.5%	9.9%
5	3.8%	5.0%	5.9%

In Table 3, for each class separately,
the two-year and three-year credibilities
are compared to the one-year credibility.

For Class 1, the ratio of the two-year to one-year credibility is: 6.8%/4.6% = 1.48.

As shown in Table 3 of Bailey-Simon:

	Relative Credibility		
Class	One-Year	Two-Year	Three-Year
1	1.00	1.48	1.74
2	1.00	1.33	1.51
3	1.00	1.33	1.57
4	1.00	1.20	1.39
5	1.00	1.32	1.55

These credibilities go up
<u>much less than linearly</u>
as the number of years of data increase.

Bailey-Simon gives possible reasons:

1. Risks entering and leaving the class.

2. An individual insured's chance for an accident changes from time to time within a year and from one year to the next.
(Shifting Risk Parameters.)

3. The risk distribution of individual insureds has a marked skewness reflecting varying degrees of accident proneness.

4. The Buhlmann Credibility formula,
$Z = N / (N+K)$,
increases less than linearly with N.
(form Hazam's Discussion.)

2.38. (9, 11/02, Q.47) a. (1.5 points)
Given the following data, calculate the credibilities for 1-year and 2-year claim free periods.
A represents 3 or more years since the most recent accident.
X represents 2 years since the most recent accident.
Y represents 1 year since the most recent accident.
B represents 0 years since the most recent accident.

	Earned Car Years	Earned Premium at Present Class B Rates	Number of Claims
A	50,000	$5,500,000	5,000
X	6,500	$682,500	1,000
Y	5,000	$535,000	850
B	4,500	$490,500	900
TOTAL	66,000	$7,208,000	7,750

9, 11/02, Q.47. (a) Overall the claim frequency on a premium basis is: 7750 / 7208 = 1.0752.

For two or more years claim free (A + X), claim frequency is:
(5000 + 1000) / (5500 + 682.5) = 0.9705.

1 - Z = 0.9705 / 1.0752. ⇒ Z = **9.7%.**

For one or more years claim free (A + X + Y), claim frequency is:
(5000 + 1000 + 850) / (5500 + 682.5 + 535) = 1.0197.

1 - Z = 1.0197 / 1.0752. ⇒ Z = **5.2%.**

9, 11/02, Q.47 b. (0.5 point)
Give two possible reasons that the 2-year credibility is less than 2 times the 1-year credibility.

9, 11/02, Q.47 (b)

1. Individual insured's chance for an accident changes from time to time within a year or from one year to the next.
2. Insureds are entering or leaving the class.
3. Individuals' accident propensities in a class vary and are markedly skewed.
4. The Buhlmann Credibility formula is less than linear.

P. 123 Alternate Way to Estimate 1-Year Cred.:

Bailey-Simon also backs out a one-year credibility by comparing the observed frequency in the prior year of those who were not claims-free (Merit Rating Group B) to their observed frequency in the subsequent year.

Assume that the overall frequency is Poisson with mean λ.

$f(0) = e^{-\lambda}$.

Let x = mean number of claims for those who were not claim free (Group B).

$\lambda = 0\ e^{-\lambda} + x\ (1 - e^{-\lambda})$. $\Rightarrow x = \lambda / (1 - e^{-\lambda})$.

For example, as shown in Table 1, for Class 1 the observed overall frequency per <u>exposure</u> is:
288,019 / 3,325,714 = 0.0866.

Then the mean number of claims for those who were not claim free (Group B) is:
$\lambda / (1 - e^{-\lambda}) = 0.0866 / (1 - e^{-0.0866}) = 1.044.$

Thus Group B has a frequency relative to average within Class 1 of:
$1 / (1 - e^{-\lambda}) = 1 / (1 - e^{-0.0866}) = 12.05.$

However, based on its relative premium based frequency, in Table 1 we have an estimated modification for Group B in Class 1 of:
2.190 / 1.484 = 1.476.

Thus, 1.476 = (12.05) Z + (1)(1 - Z). ⇒
Z = (1.476 - 1) / (12.05 - 1) = 4.3%.

This is similar to the 4.6% one-year credibility for Class 1 that is shown in Table 2
and based on the claims-free discount.

Let λ = the mean claim frequency (per exposure) for the class.

M = relative premium based frequency for risks with one or more claims in the past year.

Then, $M = Z / (1 - e^{-\lambda}) + (1 - Z)(1)$.

$$\Rightarrow Z = \frac{M - 1}{1 / (1 - e^{-\lambda}) - 1} = (M - 1)(e^{\lambda} - 1).$$

Here are the similar results for all of the classes:

Class	Mean Freq. Overall	Mean Freq. For B	Prior Rel. For B	Subseq. Rel. For B	One Year Cred.	Table 2 One Year Cred.
1	8.66%	1.044	12.05	1.476	4.3%	4.6%
2	12.05%	1.061	8.81	1.307	3.9%	4.5%
3	14.24%	1.073	7.53	1.362	5.5%	5.1%
4	16.21%	1.083	6.68	1.247	4.3%	7.1%
5	10.96%	1.056	9.63	1.302	3.5%	3.8%

There is a reasonable match between the credibilities from looking at Group B and those from the claims-free discount, with the exception of Class 4.

These two different techniques are expected to produce similar but somewhat different results, neither of which is equal to the least squares credibility.

Standard Method	Alternative Method
actual past claim frequency	theoretical past claim frequency
nonparametric	Poisson Distribution
claim frequency to premiums	claim frequency to exposures
claims-free risks	not claims-free risks
1, 2, and 3 year credibilities	one year credibility

2.11. (4.5 points) Based on Bailey and Simon's paper "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" and the information given below, calculate the credibilities that can be assigned to the experience of a single private passenger car from each of the following two groups:

a. (1.5 points) The group of risks that have been claim free for one (1) or more years.

b. (1.5 points) The group of risks that have been claim free for no (0) years.

c. (1.5 points) Discuss why the techniques in parts (a) and (b) usually give different estimates of the credibility of one year of data.

Group	Number of Years Claim Free	Earned Car Years	Earned Premium at Present B Rates	Number of Claims Incurred
A	3 or more	185,000	225,000,000	18,200
X	2	12,000	15,000,000	1,400
Y	1	15,000	20,000,000	2,200
B	0	28,000	40,000,000	5,200
Total		240,000	300,000,000	27,000

2.11. a. The overall premium based frequency is: 27,000 / 300 = 90.

The premium based frequency for those claims-free for 1 or more years (A + X + Y) is: (18,200 + 1400 + 2200) / (225 + 15 + 20) = 83.85.

1 - Z = 83.85 / 90. $\Rightarrow$ Z = **6.8%**.

b. The premium based frequency for those claims-free for 0 years (B) is: 5200 / 40 = 130.
$\Rightarrow$ Modification for Group B is: 130 / 90 = 1.444.

Overall frequency per exposure is:
27,000 / 240,000 = 0.1125.

Given the Poisson assumption, the relative observed frequency for those who had at least one claim is:
$1 / (1 - e^{-\lambda}) = 1 / (1 - e^{-0.1125}) = 9.398.$

Thus we must have: 1.444 = Z 9.398 + (1 - Z) 1.
$\Rightarrow$ Z = (1.444 - 1) / (9.398 - 1) = **5.3%**.

c. As always with finite data sets we have random fluctuation.
In addition, each technique makes assumptions and approximations.
The premium based frequencies only approximately adjust for the maldistribution of the Groups by territory.
In part (b) we had to make use of a Poisson assumption.

However, more fundamentally, we are measuring two somewhat different things.

In part (a), we are attempting to back out the weight that would have done best in predicting the future experience of those insureds who had no claims this year (A + X + Y).

In part (b), we are attempting to back out the weight that would have done best in predicting the future experience of those insureds who had at least one claims this year (B).

The Bayes Analysis estimates for different groups, those with 0 claims, those with 1 claim, those with 2 claims, etc. usually do <u>not</u> lie upon a straight line. (Only in special cases such as
the Gamma-Poisson,
are the Bayes estimates along a straight line, and thus Buhlmann Credibility equals Bayes Analysis.)

Thus the optimal weight to use in each of these situations would be different.

<u>Comment</u>: The Buhlmann credibility is the slope of the weighted least squares line fit to the Bayes Estimates as function of the observations.
Thus we would expect the estimates in parts (a) and (b) to differ from each other as well as the Buhlmann credibility.

P. 143 Appendix II:

Let

y = mean frequency of those who have had at least one claim in the last year.

overall mean = 0 f(0) + y {1 - f(0)}.

⇒ y = (overall mean) / {1 - f(0)}.

Let R = the ratio of the actual losses to the expected losses.

Then R = 1 / {1 - f(0)}.

Then the mod is: Z R + 1 - Z.

If the frequency is Poisson, then $f(0) = e^{-\lambda}$,
and for those who have at least one accident
$R = 1 / (1 - e^{-\lambda})$.

For example, if $\lambda = 0.0866$,
then $R = 1 / (1 - e^{-0.0866}) = 12.055$.

If instead the frequency is Negative Binomial parameterized as per Bahnemann,
then $f(0) = (1-p)^r$,
and for those who have at least one accident:

$$R = \frac{1}{1 - (1-p)^r}.$$

P. 144 The Discussion by William J. Hazam:

William J. Hazam, CAS President in 1968.
Leroy J. Simon, CAS President in 1971.

Bailey-Simon divide claims by premiums at the Group B rate, in order to get frequencies to compare.

Hazam points out:
"that a premium base eliminates maldistribution only if
(1) high frequency territories are also high premium territories and
(2) if territorial differentials are proper."

Buhlmann Credibility formula: $Z = N / (N + K)$.

For large K, the credibility increases only slightly less than linearly.

While this does not explain the behavior observed by Bailey-Simon, it is one reason why the credibilities would go up less than linearly.

Hazam mentions that many Merit Rating plans in the U.S. use moving traffic violations in addition to claims.

The addition of this useful information allows one to better distinguish between insureds within the same class, and therefore justifies larger credits and larger surcharges than when using just claims history.

The amount of credibility depends as well on how refined the class plan is.

The more homogeneous the classes, the less need there is for Merit Rating, and the smaller the credibility assigned to the data of an individual insured.

P. 159 **The 3 Conclusions of Bailey-Simon:**

(1) The experience for one car for one year has significant and measurable credibility for experience rating.

(2) In a highly refined private passenger rating classification system which reflects inherent hazard, there would not be much accuracy in an individual risk merit rating plan, but where a wide range of hazard is encompassed within a classification, credibility is much larger.

(3) If we are given one year's experience and add a second year we increase the credibility roughly two-fifths.
Given two years' experience, a third year will increase the credibility by one-sixth of its two-year value.

2.17a (1 point)

You are given the following private passenger automobile results for the state of Fremont.

Class	Claim Frequency per Car Year	One-year Credibility	Three-year Credibility
1	0.07	0.05	0.10
2	0.08	0.09	0.17
3	0.09	0.08	0.17

For which class do its insured have more stable expected claim frequencies over the three year period?

Assume that there is no change in the exposures in each class during the three years and that the risk distribution in each class is not markedly skewed. Explain your answer.

2.17a. Bailey & Simon give 3 reasons why the credibilities increase less than linearly with number of year of data. The question has eliminated two of these reasons; the one that is left is shifting risk parameters. The faster parameters shift over time, the greater the effect of lowering the ratio of 3-year to 1-year credibility.
The ratios of three year to one year credibilities are for the given classes: 2, 1.9, and 2.1.
Thus Class 2 has been most affected by shifting risk parameters over time and Class 3 the least.
Thus, the insureds in Class **3** have more stable expected claim frequencies from year to year.

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There is an <u>inherent problem</u> in the use of Class 4 (Unmarried Owner or Principal Operator under 25) in the claims-free analysis of Bailey-Simon, which applies to a lesser extent, to Class 5 (Married Owner or Principal Operator under 25).

The key point is that one cannot have three clean years of experience unless one has been licensed for at least three years.

Most of the drivers with less than 3 years experience are in Class 4.
Class 4 has a considerable percent of drivers who have less than three years of driving experience.

Those risks with one year of experience go into Merit Rating Class Y (clean for one year) if they are clean, and Merit Rating Class B (clean for less than one year) if they are not.

We expect drivers with less than
3 years of experience
to be worse than the average for Class 4.

Merit Rating Class A (clean for three years) has none of those with less than 3 years of experience.

Merit Rating Class A has better experience than average just due to this.

Thus when we compare Merit Rating Class A to the average of driving Class 4, which includes many inexperienced drivers, the resulting Bailey-Simon credibility for three years of data is overstated.

The same is true to a lesser extent for the Bailey-Simon credibility for two years of data.

2.14. (3 points) An insurance company has a private passenger auto book of business. There is the following claims experience for Class 1 in State X:

Territory	Earned Premium at Present Rates Prior to Merit Rating	Earned Car Years	Number of Claims
A	$15,000,000	20,000	800
B	$25,000,000	28,000	1250
C	$30,000,000	30,000	1300
D	$25,000,000	23,000	1100
E	$20,000,000	17,000	800
Total	$115,000,000	118,000	5250

You will be trying to determine the credibility of a single private passenger car for Class 1 in State X, by comparing the experience of those who are claims-free for various periods of time to the experience of all cars in Class 1 in State X. Which ratio would be more appropriate to use in this analysis:

$$\frac{\text{Number of Claims}}{\text{Dollars of Earned Car Years}} \text{ or } \frac{\text{Number of Claims}}{\text{Dollars of Earned Premiums}}\text{?}$$

Justify your selection. Is there some other ratio that you would use instead of these two?

2.14. Bailey-Simon uses $\frac{\text{Number of Car Years}}{\text{Dollars of Earned Premiums}}$,

in order to adjust for the maldistribution that would result from low frequency territories having a larger portion of insureds who are claims-free.

It would be better to use premiums, provided the high rated territories have higher frequency and provided the territory relativities are correct.

Territory	Average Rate	Relative to Average	Frequency per Car-Year	Relative to Average
A	$750	0.769	4.00%	0.899
B	$893	0.916	4.46%	1.002
C	$1000	1.026	4.33%	0.973
D	$1086	1.114	4.78%	1.074
E	$1176	1.206	4.70%	1.056
Total	$975	1.000	4.45%	1.000

There is a tendency for the higher rated territories to have higher frequencies.

However, the relative average rates have a much wider spread than the relative average frequencies.

Thus the average premiums largely reflect differences in severity and/or reflect incorrect territory relativities in the current rates.

Using for each subgroup (0 years claims-free, 1 year claims-free, 2 years claims free, etc.)
$\frac{\text{Number of Claims}}{\text{Dollars of Earned Premiums}}$ would adjust for the differences in frequency by territory, but would significantly over-adjust due to whatever is causing the wider differences in average premium.

Using $\frac{\text{Number of Claims}}{\text{Number of Earned Car Years}}$ would <u>not</u> adjust for the differences in frequency by territory.

In this case, the other reasons for differences in average premiums seem to have a bigger effect than differences in frequency.

Thus on balance I would prefer to use

$$\frac{\text{Number of Claims}}{\text{Number of Earned Car Years}}$$

rather than $\frac{\text{Number of Claims}}{\text{Dollars of Earned Premiums}}$.

We want to adjust for the different mixes of territory for the subgroups,
due to the different frequencies by territory

If possible, it would probably be better to use for each subgroup (0 years claims-free,
1 year claims-free, 2 years claims free, etc.):

$$\frac{\text{Number of Claims}}{\sum\limits_{\text{terrs}} \text{(caryears subgroup in terr.) (rel. freq. within terr. to class)}}.$$

In this case, the relative frequencies for the territories within Class 1 are:
0.899, 1.002, 0.973, 1.074, 1.056.

Assume that the subgroup that is claim free for at least 3 years has exposures within Class 1 by territory: 17,700, 24,500, 26,300, 19,900, 14,800.

Then the above denominator would be:
(0.889)(17,700) + (1.002)(24,500)
+ (0.973)(26,300) + (1.074)(19,900)
+ (1.056)(14,800) = 102,876.

This is less than the sum of exposures for this subgroup of 103,200, reflecting the somewhat higher proportion of low frequency territories in this subgroup than in all of Class 1.

<u>Comment</u>: Similar to 8, 11/12, Q.6.

2.18. (2 points) For a specific class, the following data shows the experience of a merit rating plan.

Merit Rating	Number of Accident-Free Years	Earned Premium at Present B Rates	Number of Incurred Claims
A	3 or More	$2400 million	12,000
X	2	$200 million	1200
Y	1	$220 million	1400
B	0	$380 million	2600
	Total	$3200 million	17,200

The base rate (for Merit Rating B) is $800 per exposure for this class. Calculate the appropriate premium for an exposure that is accident free for one or more years.

2.18. The indicated rate compared to average for those who are one or more years claims free is:

$$\frac{(12000 + 1200 + 1400) / (2400 + 200 + 220)}{17{,}200/3200}$$

= 5.1773 / 5.375 = 0.9632.

The indicated rate compared to average for those who are not claims free is:

$$\frac{2600/380}{17{,}200/3200} = 6.8421 / 5.375 = 1.2729.$$

Thus the appropriate premium for an exposure that is accident free for one or more years is:
(0.9632/1.2729) ($800) = **$605.36**.

Alternately, (5.1773/6.8421) ($800) = **$605.35**.

Comment: Similar to 8, 11/14, Q.5.